Aerotaxis problem

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Abstract

We study the existence and uniqueness of the solutions in a model describing the evolution of density of bacteria and oxygen dissolved in water filling a capillary

$$\begin{cases} u_t = (u_x - uE(p)_x)_x, \\ p_t = p_{xx} - E(p)u, \\ u(x,0) = u_0(x) \geq 0, \quad p(x,0) = p_0(x) \geq 0. \end{cases}$$

Here p and u are unknown function, $E(p)$ is a given energy function. In biological interpretation $p,\ u$ are respectively the density of the oxygen and bacteria.

The steady state is a stationary solution of nonlinear and nonlocal problem which depends on the energy function and contains two parameters: total mass of the colony of bacteria and the concentration (or flux) of oxygen at the end of capillary. The existence and uniqueness of stationary solutions depend on relations between these parameters and the maximum of energy function. The existence of global in time solution is proved and its convergence to the stationary solution.