Blow-up profiles for the parabolic-elliptic Keller-Segel system in dimensions $n \geq 3$

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Abstract

We study the blow-up asymptotics of radially decreasing solutions of the parabolic-elliptic Keller-Segel system in space dimensions $n \geq 3$, in a ball or in the whole space. In view of the biological background of this system and of its mass conservation property, blowup is usually interpreted as a phenomenon of concentration or aggregation of the bacterial population. Understanding the asymptotic behavior of solutions at the blowup time is thus meaningful for the interpretation of the model.

Under mild assumptions on the initial data, for $n \geq 3$, we show that the final profile satisfies

$$C_1|x|^{-2} \leq u(x,T) \leq C_2|x|^{-2},$$

with convergence in $L^1(B_R)$. This is in sharp contrast with the two-dimensional case, where solutions are known to concentrate to a Dirac mass at the origin (plus an integrable part) – cf. Herrero-Velázquez, Senba-Suzuki. For any radial decreasing blowup solution, we also obtain the refined space-time estimate

$$u(x,t) \leq \Big(\frac{1}{u(0,t)} + C|x|^2\Big)^{-1},$$

hence

$$u(x,t) \leq C(T-t+|x|^2)^{-1}$$

for type I blowup solutions. Previous work (Herrero-Medina-Velázquez, Guerra-Peletier, Giga-Mizoguchi-Senba) had shown that radially decreasing self-similar blowup solutions (which satisfy the above estimates) exist in dimensions $n \geq 3$ and do not exist in dimension 2. Our results thus show that the profiles displayed by these special solutions actually correspond to a much more general phenomenon.