Global existence for some chemotaxis-haptotaxis models

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Abstract

We study chemotaxis-haptotaxis models of the form

$$\begin{cases} \partial_t c = \nabla \cdot (D(c,v)\nabla c) - \nabla \cdot (\psi(v) c \nabla v) - \nabla \cdot (f(c,l) c \nabla l) & \\ \qquad +\mu_c c(1-c-v), & x \in \Omega, t > 0, \\ \partial_t v = -\delta c v + \mu_v (1-c-v), & x \in \Omega, t > 0, \\ \partial_t l = \Delta l - l + cv, & x \in \Omega, t > 0, \end{cases} \tag{0.1}$$

endowed with homogeneous Neumann boundary conditions, where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary and $n \leq 3$.

Models of type 0.1 appear in the context of tumor invasion, where c and v denote the densities of tumor cells and extracellular matrix, respectively, and l is the concentration of a chemoattractant (e.g. proteolytic remainders).

In this talk we will explain a method to prove the global existence of a weak solution to 0.1. The method relies on the construction of an entropy-type functional for some regularized problems approximating 0.1. The functional allows to obtain appropriate compactness properties for solutions of these approximate problems and to construct a global weak solution to 0.1. In particular, the method yields the global existence in dimension $n = 3$ in the case

$$D(c,v) = \frac{1}{1+cv}, \quad \psi(v) = \frac{v}{1+v}, \quad f(c,l) = \frac{1}{1+cl}$$

and we will discuss to what extent the method can be applied for other cases of 0.1. In addition, we show that a variant of the method can also be used for a model coupling a haptotaxis equation to two ODEs.

This talk is mainly based on the joint works [1, 2] with C. Surulescu, A. Uatay, and M. Winkler.

References

[1] C. Stinner, C. Surulescu, and A. Uatay, *Global existence for a go-or-grow multiscale model for tumor invasion with therapy.* Mathematical Models and Methods in Applied Sciences **26**, No. 11, 2163–2201 (2016).

[2] C. Stinner, C. Surulescu, and M. Winkler, *Global weak solutions in a PDE-ODE system modeling multiscale cancer cell invasion.* SIAM Journal on Mathematical Analysis **46**, No. 3, 1969–2007 (2014).