ON TOPOLOGICAL CLASSIFICATION OF MORSE-SMALE SYSTEMS ON SURFACES

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The results were obtained in collaboration with V. Grines and S. van Strien.

One of the main objectives in the field of dynamical systems is to obtain a classification in terms of their dynamics. Such a classification was achieved successfully in the one-dimensional setting. For example, two circle diffeomorphisms $f, f' : S^1 \to S^1$ are called topologically conjugate if there exists an orientation preserving homeomorphism $h : S^1 \to S^1$ so that $hf = fh$. In the 1880's, Poincare [2] showed that if these two diffeomorphisms are topologically conjugate then they have the same rotation number. Moreover, for two transitive homeomorphisms f, f' the condition that their rotation numbers are the same is both necessary and sufficient for the topological conjugacy. In 1932, Denjoy [1] improved this result by showing that if the diffeomorphism f is C^2 and have no periodic orbits then it is transitive.

If f, f' have periodic orbits and each of these periodic orbits is hyperbolic (such a diffeomorphism is called *Morse-Smale*), then a necessary and sufficient condition for these circle diffeomorphisms to be topologically conjugate is that their rotation numbers are the same and that they have the same number of periodic attractors. Moreover, if one chooses a rotation compatible permutation on a finite number of points on the circle, then this data corresponds to a Morse-Smale diffeomorphism. For *non-invertible* Morse-Smale maps of the circle or the interval one has a similar situation: it's so-called *kneading map* (describing itineraries of its turning points) is (essentially) a complete topological invariant and, moreover, each admissible kneading map corresponds to a map of the circle.

The *aim* of this paper is to establish a corresponding classification in the setting of Morse-Smale diffeomorphisms on *closed orientable surfaces*, replacing a finite number of points on a circle by a finite number of annuli on tori.

Theorem A (Classification by finite amount of data) *Let M be a closed orientable surface and $f\colon M \to M$ be an orientation preserving Morse-Smale diffeomorphism. Then one can assign to f a scheme S_f or a decomposed scheme consisting of a finite amount data (given by a finite union of tori, and the homotopy type of certain annuli in these tori), in such a way that $f\colon M \to M$ and $f'\colon M' \to M'$ are topologically conjugate if and only if S_f is equivalent to $S_{f'}$.*

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Theorem B (Realisation) *Each abstract scheme S corresponds uniquely to an orientable closed surface M and an orientation preserving Morse-Smale diffeomorphism $f\colon M \to M$.*

References

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[2] H. Poincaré, Mémoire sur les courbes définies par uneéquation différrentielle I, II, III, IV. J. Math. Pures et Appl. (1881), 7, 375-422, (1882), 8, 251-286, (1885), 1, 167-244, (1886), 2, 151-217.

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