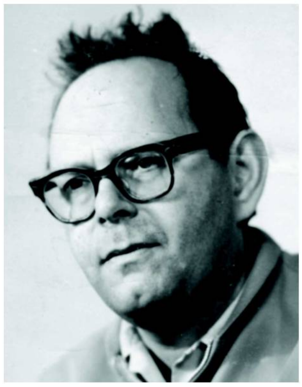
Warsaw Center of Mathematics and Computer Science

Banach Center
INSTITUTE OF MATHEMATICS PAS

IV Spring School of Analysis in memory of Aleksander Pełczyński



Dirichlet Series

5-8.04.2018 Będlewo

Andreas Defant (Oldenburg)

Dirichlet series and holomorphic functions in high dimensions

Kristian Seip (Trondheim)

Hardy spaces of Dirichlet series and the Riemann zeta function

Eero Saksman (Helsinki)

On multiplicative chaos measures and the boundary behaviour of Dirichlet series of Hardy spaces

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1 Andreas Defant

We take as starting point the work of Harald Bohr over a hundred years ago. There he stated a very concrete problem about convergence of Dirichlet series: *What is the largest width among all possible strips in the complex plain on which Dirichlet series* $\sum_n a_n n^{-s}$ *converge uniformly but not absolutely?*

In order to solve this so-called 'absolute convergence problem' Bohr realized, with a simple, genial idea, that Dirichlet series and formal power series were intimately related through prime numbers. Let us shortly describe this relationship. A formal power series is of the form $\sum c_\alpha z^\alpha$, where the α's are multi-indices $(\alpha_1, \ldots, \alpha_n)$ of arbitrary length and for a given sequence $z = (z_n)_n$ and some such α, $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$. Now, given some natural number n, we take its decomposition into prime numbers (which we denote by p_k) $n = p_1^{\alpha_n} \cdots p_k^{\alpha_k}$. Then, by the unicity of this decomposition, to each n corresponds a unique α and vice-versa. In other words, to coefficients $(a_n)_n$ corresponds (by defining $c_\alpha := a_{p^\alpha}$) a unique family of coefficients (c_α), and to each Dirichlet series $\sum a_n n^{-s}$ corresponds a unique power series $\sum c_\alpha z^\alpha$. This correspondence, that we will call *Bohr's tranform* is one of the major issues.

Bohr's new ideas were undertaken and elaborated by Bohnenblust and Hille, who finally solved the absolute convergence problem 18 years later showing that the largest possible strip (as described above) in fact is 1/2. These ideas are the seed from which all what we intend to present grows. Long time later Bohr's problem happened to be closely related with other interesting problems and aspects in different areas of analysis. We are going to focus on two of them: holomorphic functions on infinite dimensional polydiscs (in functional analysis) and Hardy spaces on the infinite dimensional torus (in harmonic analysis).

2 Kristian Seip

We concentrate mainly on the two topics

(1) Gál-type GCD sums and the growth of the Riemann zeta function (2) Contractive inequalities in Hardy spaces and pseudomoments of the Riemann zeta function

We make an excursion into number theory. We explain a bit about the major obstacles in the theory of $\zeta(s)$, including what remains even if assuming the Riemann hypothesis. We include some basic information about $\zeta(s)$. One of the virtues of the second topic, is that it illustrates the two-way interaction between number theory and Hardy spaces of Dirichlet series: it shows how an interesting number theoretic problem has led to a still open problem in the unit disc—a problem reminiscent of problems studied by Hardy and Littlewood.

3 Eero Saksman

We consider some basic questions related to the boundary behaviour of elements in the Hardy spaces of Dirichlet series. Our main goal is to give an idea of the recent resolution by Adam Harper of the so-called embedding problem. For that purpose we shall introduce the basic theory of multiplicative chaos measures. At the end we also explain how these measures can be used to describe the statistics of the Riemann zeta function over the critical line.

Conference cost 120 EUR

For registration and information

www.impan.pl/en/activities/banach-center/conferences/18-vischoolofanalysis