18th Workshop: Noncommutative Probability, Operator Algebras, Random Matrices and Related Topics, with Applications

LIST OF ABSTRACTS

15-21.07.2018, Będlewo

1. Hélène Airault (Université de Picardie Jules Verne, Lamfa, CNRS 7352, Amiens- France)

   Matrix operators on symmetric Kähler domains.

   Abstract: Let D be the Kähler manifold of $n \times n$ symmetric matrices Z such that $I - \overline{Z}Z > 0$ and the holomorphic representation $T_g F(z) = k'_g(Z)^\gamma F(k_g(z))$ of the group $G = Sp(2n)$. A suitable basis of the real vector space $\mathcal{G}$, the Lie algebra of G, can be choosen such that the infinitesimal holomorphic representation relatively to this basis is given by the Hua matrix operators

   $$\begin{aligned} \mathcal{V}_{\mathfrak{t}} &= (Z\partial_Z - \partial_Z Z) + i\,(Z\partial_Z + \partial_Z Z) \qquad (i) \\ \mathcal{V}_{\mathfrak{p}} &= (\partial_Z - Z\partial_Z Z) + i\,(\partial_Z + Z\partial_Z Z) \qquad (ii) \end{aligned} \quad \text{where} \quad \partial_Z = \frac{1}{2}((1+\delta_{ij})\frac{\partial}{\partial z_{ij}})$$

   Traces in the matrix products $\mathcal{V}_{\mathfrak{t}}\overline{\mathcal{V}_{\mathfrak{t}}}$ and $\mathcal{V}_{\mathfrak{p}}\overline{\mathcal{V}_{\mathfrak{p}}}$ give the Laplace-Beltrami and Ornstein-Uhlenbeck operators on D. Similar results stay valid for Kähler submanifolds $\mathcal{M}$ of D when the coefficients of the matrix Z depends linearly on p independent coefficients with $p < n(n+1)/2$. We determine the invariant measures for the Laplacian and Ornstein-Uhlenbeck operators on $\mathcal{M}$.

   [1] Hélène Airault, Abdelhamid Boussejra; Lifted infinitesimal holomorphic representation for the n-dimensional complex hyperbolic ball and for Cartan domains of type I. Bull. Sci. Math. 137 (2013), no. 7, 923 -967.

   [2] Hélène Airault, Abdelhamid Boussejra; Preprint (Novembre 2017).

2. Maryam Alshehri (Swansea University)

   Particle-hole duality in the continuum and determinantal point processes

   Abstract: Let X be an underlying locally compact Polish space equipped with a Borel measure σ. Let $K(x,y) : X^2 \to \mathbb{C}$ and let K denote the integral operator in $L^2(X,\sigma)$ with integral kernel $K(x,y)$. A point process μ on X is called determinantal with the correlation operator K if the correlation functions of μ are given by $k^{(n)}(x_1,\dots,x_n) = \det[K(x_i,x_j)]_{i,j=1,\dots,n}$. If the operator K is self-adjoint, a determinantal point process with correlation operator K exists if and only if K is locally trace-class and $0 \leqslant K \leqslant 1$. Each determinantal point process with a Hermitian correlation kernel can be understood as the (spectral measure of) the particle density $\rho(x) = \partial_x^\dagger \partial_x$ $(x \in X)$, where the operator-valued distributions $\partial_x^\dagger$, ∂_x come from a gauge-invariant quasi-free representation of the canonical anticommutation relations (CAR). If the space X is discrete and divided into two disjoint parts, X_1 and X_2, by exchanging particles and holes on the X_2 part of the space, one obtains from a determinantal point process with correlation kernel K a determinantal point process with correlation kernel $\widehat{K} = KP_1 + (1-K)P_2$, where P_i is the orthogonal projection onto $L^2(X_i,\sigma)$. In particular, the operator $\widehat{K}$ is J-self-adjoint. In the case where the space X is continous, a direct procedure of swapping particles and holes makes no sense. Nevertheless, we prove that it is possible to carry out such a procedure by swapping creation operators $\partial_x^\dagger$ with annihilation operators ∂_x on the X_2 part of the space. This leads to a quasi-free representation of CAR and the corresponding particle density is a determinantal point process with correlation operator $\widehat{K}$, which is J-self-adjoint.

3. Hiroshi Ando (Chiba University)

Structure of bicentralizer algebras and inclusions of type III factors

Abstract: Let M be a type III_1 factor with a faithful normal state φ. The *bicentralizer* of (M, φ), denoted by $\mathrm{B}(M, \varphi)$ is the von Neumann subalgebra of M consisting of all $a \in M$ for which $\lim_n \|ax_n - x_n a\|_\varphi = 0$ holds for every bounded sequence $(x_n)_{n=1}^\infty$ in M which satisfies $\lim_n \|x_n\varphi - \varphi x_n\| = 0$. It was introduced by Connes in his strategy to prove the uniqueness of the injective type III_1 factor. Namely he showed that an injective type III_1 factor with the trivial bicentralizer is isomorphic to the Araki–Woods factor, which was affimatively solved by Haagerup. Whether every type III_1 factor with separable predual has trivial bicentralizer is an open problem. Thanks to Haagerup's characterization of type III_1 factors with trivial bicentralizers, the bicentralizer problem is still of importance in the structure theory of type III_1 factors. Connes also showed that the bicentralizers are independent of φ up to a canonical isomorphism. Recently Haagerup observed that the idea of Connes' isomorphism between bicentralizers can be enhanced to construct a canonical flow β^φ on $\mathrm{B}(M, \varphi)$ which has interesting properties. Later, the flow was independently discovered by Marrakchi. We generalize the idea to the relative setting to investigate the structure of the relative bicentralizer algebra (defined by Masuda) $\mathrm{B}(N \subset M, \varphi)$ for inclusions of von Neumann algebras with normal expectation where N is a type III_1 subfactor and $\varphi \in N_*$ is a faithful state. We construct a canonical flow $\beta^\varphi : \mathbb{R}_+^* \curvearrowright \mathrm{B}(N \subset M, \varphi)$ on the relative bicentralizer algebra and we show that the W*-dynamical system $(\mathrm{B}(N \subset M, \varphi), \beta^\varphi)$ is independent of the choice of φ up to a canonical isomorphism. We then discuss the relationship between the properties of the flow and the structure of the inclusion $N \subset M$. This is joint work with Uffe Haagerup, Cyril Houdayer and Amine Marrakchi.

4. Nicolas Behr (IRIF, Université Paris Diderot (Paris 07), France)

Explicit formulae for all higher-order lacunary generating functions of the two-variable Hermite polynomials

Abstract: Reporting on recent joint work [1] with G.H.E. Duchamp (Paris 13) and K.A. Penson (Paris 06), for a sequence $P = (p_n(x))_{n=0}^\infty$ of polynomials $p_n(x)$, we study the K-tuple and L-shifted lacunary generating functions $\mathcal{G}_{K,L}(\lambda; x) := \sum_{n=0}^\infty \frac{\lambda^n}{n!} p_{n\cdot K+L}(x)$, for $K = 1, 2 \ldots$ and $L = 0, 1, 2 \ldots$. We establish an algorithm for efficiently computing $\mathcal{G}_{K,L}(\lambda; x)$ for generic polynomial sequences P. This procedure is exemplified by application to the study of Hermite polynomials, whereby we obtain closed-form expressions for $\mathcal{G}_{K,L}(\lambda; x)$ for arbitrary K and L, in the form of certain infinite series involving generalized hypergeometric functions. Our technique reproduces all the results previously known in the literature.

[1] Behr, N., Duchamp, G. H., and Penson, K. (2018). Explicit formulae for all higher order lacunary generating functions of Hermite polynomials, in preparation.

5. Alexander Belton (Department of Mathematics and Statistics, Lancaster University)

Preservers for classes of positive matrices

Abstract: It is a simple consequence of the Schur product theorem that the class of positive semidefinite matrices is preserved by the entrywise application of an arbitrary absolutely monotonic function. As shown by work of Schoenberg, the converse is also true: a function which preserves positive semidefinite matrices of any size is necessarily absolutely monotonic. The situation is more complex for matrices of a fixed size, or when the class of matrices under study has some additional structure. This talk will address the former question and some cases of the latter, including Hankel matrices and totally non-negative matrices. This is joint work with Dominique Guillot, Apoorva Khare and Mihai Putinar.

[1] A. Belton, D. Guillot, A. Khare and M. Putinar, Matrix positivity preservers in fixed dimension. I, *Adv. Math.* 298 (2016) 325–368.

6. Abdelhamid Boussejra (Department of Mathematics, University Ibn Tofail, Kenitra - Morocco)

   On the L^p-range of the Poisson transform on Riemannian Symmetric Spaces

   Abstract: In this talk we shall give characterizations of the L^p-range of the Poisson transform P_λ on Riemannian Symmetric Spaces. In the rank one case we will show that for a non-zero real λ , the Poisson transform is a bijection from the space of L^2 functions on the boundary (respectively L^p) onto a subspace of eigenfunctions of the Laplacian satisfying certain L^2-type norms (respectively Hardy-type norms). The proof uses techniques of singular integrals on the boundary viewed as a space of homogeneous type in the sense of Coifman and Weiss. In the second part of this talk, we shall give a characterization of L^p-Poisson integrals on homogeneous line bundles on bounded symmetric domains.

7. Marek Bożejko (IM PAN)

   Generalized Gaussian Processes and positive definite functions on permutation (Coxeter) groups

   Abstract: We present main examples of Generalized Gaussian Processes (GGP) and relations with harmonic analysis on permutation and sign-permutation grous as well others Coxeter groups. Connections with free probability and Thoma characters on $S(\infty)$ and orthogonal polynomials will be also done.

8. Wojciech Bruzda (Jagiellonian University)

   Excess of a matrix and generalized Bell inequalities

   Abstract: We recall the standard definition of excess of a Hadamard matrix and propose some generalizations of this notion. We study the bounds of excess for several classes of matrices: unitary (complex Hadamard), hermitian, circulant, etc. A connection between the excess and Bell inequalities (CHSH) is provided. We show a one-to-one relation between the calculation of classical value of a Bell inequality and generalized excess of a given matrix. Joint work with Dardo Goyeneche, Ondrej Turek, Daniel Alsina, Karol Ĺ»yczkowski.

9. Marie Choda (Osaka Kyoiku University)

   Matrices over algebras

   Abstract: Motivated by some phenomenon in inclusion of matrix algebras, we consider matrices over algebras and UCP(unital completely positive) maps arising naturally from these matririces. We show some relations between properties of UCP maps and the data for inclusions of algebras.

10. Byoung Jin Choi (Chungbuk National University)

    Inequalities for Positive Module Operators on von Neumann Algebras

    Abstract: In this talk, we establish the Cauchy-Schwarz and Golden-Thompson inequalities for module operators, a generalization of a (noncommutative) conditional expectation, on a von Neumann algebra. We also apply these inequalities to the noncommutative Bennett inequality and a uncertainty relation, a generalization of the Schrödinger uncertainty relation, for conditional expectations. This talk is based on a joint work with Un Cig Ji and Yongdo Lim.

11. Barbara Ciesielska (Uniwersytet JagielloĹ„ski)

    Design of experiments with classical and quantum orthogonal arrays

    Abstract: In the paper of Dardo Goyenecho, Sara Di Martino, Zahra Raissi and Karol Ĺ»yczkowski several classes of quantum combinatorial designs, namely quantum Latin squares, cubes, hypercubes and the notion of orthogonality between them were introduced. A further introduced notion, quantum orthogonal arrays, generalizes all previous classes of design. I will briefly discuss it and show one of the application of orthogonal arrays, i.e. the design of experiments, showing usage of classical and quantum orthogonal arrays.

12. Vito Crismale (University of Bari)

Vacuum distribution and norm of sums of gaussian monotone operators

Abstract: We provide a recurrence formula to compute atoms and weights for the (discrete) vacuum distribution of sums of creation and annihilation operators $s_i := a_i + a_i^\dagger$ in monotone Fock space. The result is obtained in a direct way, without using monotone convolution, and exploiting some properties of palindromic polynomials. Moreover, we show the law above is a basic measure on the spectrum of the unital C^*-algebra generated by $\sum_{i=1}^n s_i$. This allows us to achieve the norm for any finite sum of gaussian operators as the right endpoint of the support of its vacuum distribution. This is a joint work with Y.G. Lu.

13. Adrian Dacko (Wroclaw University of Science and Technology)

The V-monotone independence in Noncommutative Probability

Abstract: We introduce and study a new notion of noncommutative independence, called V-monotone independence, which generalizes the monotone independence of Muraki. We investigate the combinatorics of mixed moments of V-monotone random variables and prove the Central Limit Theorem. We obtain a combinatorial formula for the limit moments and we find the solution of the differential equation for the moment generating function in the implicit form.

14. Antoine Dahlqvist (University College Dublin)

Permutation invariant random matrices and graph operads

Abstract: Consider two sequences of random matrices A_N and B_N of size N, whose respective $*$ non-commutative distributions converges as $N \to \infty$. What can be said asymptotically about their joint $*$-distribution? When the law of A_N or B_N is invariant by unitary conjugation, the latter is asymptotically described by the notion of free independence, as introduced by D. Voiculescu? In this talk, we shall consider what happens when this very assumption is dropped? We shall review recent progress addressing this question, focusing on ensembles where unitary invariance is weakened into the invariance by conjugation with permutation matrices.

[1] Guillaume Cébron, Antoine Dahlqvist and Camille Male, Universal constructions for spaces of traffics, arXiv:1601.00168.

[2] Benson Au, Guillaume Cébron, Antoine Dahlqvist, Franck Gabriel and Camille Male. Large permutation invariant matrices, work in progress.

15. Biswarup Das (University of Wrocław)

Admissibility of quantum group representations and its connection with Property (T) for quantum groups

Abstract: In 2005, P. Sołtan, while studying Bohr compactification of quantum semigroups, proposed the admissiblity conjecture for quantum group representations, which essentially means the following: All finite dimensional unitary representation of a locally compact quantum group factors through a matrix quantum group. This conjecture is still unsettled, although some partial progress were made through works of Sołtan and Daws. In this talk, we will systematically study this conjecture and as a by-product of out techniques (subject to availability of time) we will extend some results about Property (T) for quantum groups, earlier obtained by Daws, Skalski and Viselter. Based on a joint work with M. Daws and P. Salmi.

16. Antoine Derighetti (École Politechnique de Lausanne)

On the sets of uniqueness in noncommutative locally compact groups in the sense of Mareck Bozejko

Abstract: Let G be a locally compact group and $1 < p < \infty$. We show that a closed subgroup H of G is a set of p-uniqueness if and only H is locally negligible. We also obtain, in analogy with the case of sets of synthesis, an inverse projection theorem for sets of p-uniqueness.

17. Wiktor Ejsmont (Uniwersytet Ekonomiczny, Wrocław)

Free tangent law

Abstract: In this presentation we formulate a central limit theorem for the sums of free comutators and introduce a generalized tetilla distribution.

18. Gero Fendler and Michael Leinert (Heidelberg)

Convolution dominated operators on compact extensions of abelian groups

Abstract: If G is a locally compact group, $CD(G)$ the algebra of convolution dominated operators on $L^2(G)$, then an important question is: Is $\mathbb{C} \cdot 1 + CD(G)$ (or $CD(G)$ if G is discrete) inverse-closed in the algebra of bounded operators on $L^2(G)$? In this note we answer this question in the affirmative, provided G is such that one of the following properties is satisfied.

(1) There is a discrete, rigidly symmetric, and amenable subgroup $H \subset G$ and a (measurable) relatively compact neighbourhood of the identity U, invariant under conjugation by elements of H, such that $\{hU \,:\, h \in H\}$ is a partition of G.

(2) The commutator subgroup of G is relatively compact. (If G is connected, this just means that G is an IN group.)

All known examples where $CD(G)$ is inverse-closed in $B(L^2(G))$ are covered by this.

19. Uwe Franz (Université de Bourgogne Franche-Comté)

Invariant states on the Brown-Glockner-von Waldenfels algebra

Abstract: Denote by $C(U_n^{\mathrm{dual}})$ the universal C*-algebra generated by the coefficients of a $n \times n$ unitary $U = (u_{jk})_{1 \leqslant j,k \leqslant n}$. Voiculescu showed that this algebra can be equipped with the structure of a dual group and Cébron and Ulrich studied its properties from a quantum probabilistic viewpoint, see [CU16, Ulr15] and the references therein. In particular, they defined convolution products associated to the five universal notions of independence (tensor, free, monotone, boolean, and anti-monotone) for states on $C(U_n^{\mathrm{dual}})$. Cébron and Ulrich showed that there exist so-called *tensor and free Haar traces*, i.e. tracial states that are invariant under tensor or free convolution with other tracial states. In my talk I will introduce a family of automorphism groups on $C(U_n^{\mathrm{dual}})$ and I will show that for each of these automorphism groups there exists a tensor and a free Haar KMS-state, i.e. a state satisfying a KMS property that is invariant under tensor or free convolution with any other state satisfying the same KMS property. This leads to a new family of reduced versions of the Brown algebra. My talk is based on joint work with Guillaume Cébron and Michaël Ulrich.

[CU16] Guillaume Cébron, Michaël Ulrich, Haar states and Lévy processes on the unitary dual group. *J. Funct. Anal.*270 (2016), no. 7, 2769-2811.

[Ulr15] Michaël Ulrich, Construction of a free Lévy process as high-dimensional limit of a Brownian motion on the unitary group. *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* 18 (2015), no. 3, 1550018.

20. Avital Frumkin and Assaf Goldberger (TAU Israel)

Harmonic factorization of matrix'polynomials and Imanent'computations

Abstract: Let $g = gl_n(\mathbb{C})$ be the Lie algebra of all n by n matrices over the complex numbers and $G = GL_n(\mathbb{C})$ be the general linear group over the complex numbers. G acts on g by conjugation: $a \circ g = g^a = aga^{-1}$. Let $S(g)$ denote the ring of polynomials whose indeterminates are the elements of the matrices of g. The $\circ$ action of G on g induces an action on $S(g)$ we denote also by $\circ$.

Let $J_G(g) \subseteq S(g)$ be the ring of G-invariant polynomials and $H \subseteq S(g)$ (H for harmonic) is the subspace of the polynomials vanishing under all differential operators coming from G invariants. By Kostant(JAMS1963) any polynomial in $S(g)$ can be written uniquely as linear combination of harmonics with G invariants as coefficients. This is called the *harmonic factorization*. For given $f \in S(g)$ and a given diagonal matrix t, define a function $\widetilde{f^t}$ from G to the complex numbers by $\widetilde{f^t}(a) = f(a \circ t)$. The λ immanent of a matrix A defined by $Im_\lambda(A) = \sum_{\sigma \in S_n} \chi_\lambda(\sigma) \prod_i A_{i\sigma(i)}$ Where χ_λ is the S_n irreducible character of $\lambda \vdash n$. For some family of partitions λ we give the full harmonic factorization of Im_λ. By using the normality of the G-orbits of t non degenerate diagonal matrices t, and by details from Kostant (JAMS1963) and Matsuzawa (comm in algebra 1998) we show that the dimension of the space spanned by $\widetilde{Im^t_\lambda}$ where t runs over all the diagonal matrices, is closely related to the function $\sum_\eta S^2_\lambda(x^\eta)|_{x=1,1,1,..}$, summed over $\eta \vdash n$, with x specialized to 1 in each coordinate and S_λ being the Schur function of the partition λ. Some computational aspects of this algebraic results are considered.

21. Maria Elena Griseta (University of Bari Aldo Moro)

Vacuum distribution for sum of gaussian operators on weakly monotone Fock space

Abstract: In this talk we investigate the distributions of partial sums $S_m := \sum_{i=1}^m G_i$, G_i being the position operators $G_i := A_i + A_i^\dagger$ on the weakly monotone Fock space $\mathfrak{F}_{WM}(\mathcal{H})$. We establish the G_i are monotone independent, and moreover any of them has the distribution given by the Wigner semi-circle law with density $\nu(dx) := \frac{1}{2\pi}\sqrt{4-x^2}$ on $[-2, 2]$. We explicitly compute the law for the sum of two position operators, i.e the monotone convolution of the Wigner law by himself, as an absolutely continuous measure w.r.t. the Lebesgue one. Moreover, we state that for $m \geqslant 3$ the vacuum law is indeed absolutely continuous, symmetric and compactly supported on intervals of the form $[-a_m, a_m]$. Finally, for the endpoints a_m of the intervals above we achieve either a recurrence relation or a nice approximation by lower and upper bounds. From this in particular one has that the sequence $\left(\frac{a_m}{\sqrt{m}}\right)_m$ converges decreasingly to $\sqrt{2}$, as the monotone Central Limit Theorem suggests. This is a joint work with V. Crismale and J. Wysoczański.

22. Piotr M. Hajac (IMPAN)

Associated noncommutative vector bundles over the Vaksman-Soibelman quantum complex projective spaces

Abstract: By a diagonal embedding of $U(1)$ in $SU_q(n)$, we prolongate the diagonal $U(1)$-action on the Vaksman-Soibelman quantum sphere S_q^{2n+1} to the $SU_q(n)$-action on the prolongated bundle $S_q^{2n+1} \times_{U(1)} SU_q(n)$. Then, using the index pairing, we prove that the noncommutative vector bundles associated via the fundamental representation of $SU_q(n)$, for $n \in \{2, \ldots, N\}$, yield generators of the K_0-group of the C*-algebra $C(\mathbb{C}\mathrm{P}_q^N)$ of the Vaksman-Soibelman quantum complex projective space $\mathbb{C}\mathrm{P}_q^N$. Based on joint work with Francesca Arici and Mariusz Tobolski.

23. Takahiro Hasebe (Hokkaido University)

Limits of free Lévy processes at small time

Abstract: In classical probability, it is known that if the law of a Lévy process with an affine transformation converges as time goes to 0 or infinity, then the limit distribution is stable. A similar theorem holds for free Levy processes with respect to addition. However, for free Levy processes with respect to multiplication, the situation becomes different in large time, as proved by Tucci and Haagerup-Moeller. Then what happens in the small time limit? This is a joint work with Octavio Arizmendi.

24. Fumio Hiai (Tohoku University)

Convergence theorems for barycentric maps

Abstract: I first explain a theory of conditional expectations for random variables with values in a complete metric space M equipped with a contractive barycentric map β, and then give convergence theorems for martingales of β-conditional expectations. I give the Birkhoff ergodic theorem for β-values of ergodic empirical measures and provide a description of the ergodic limit function in terms of the β-conditional expectation. Moreover, I prove the continuity property of the ergodic limit function by finding a complete metric between contractive barycentric maps on the Wasserstein space of Borel probability measures on M. Finally, the large derivation property of β-values of i.i.d. empirical measures is obtained by applying the Sanov large deviation principle. This is joint work with Yongdo Lim.

25. Fumio Hiroshima (Faculty of Mathematics, Kyushu Universit)**Integral kernels of semigroup generated by a model in quantum field theory**

Abstract: This is the joint work with O.Matte [1]. The Nelson model in scalar quantum field theory is defined by Schrödinger operator $H_p = -\Delta/2 + V$ in $L^2(R_x^d)$ linearly coupled to a scalar bose field $H_i = \phi(\varphi(\cdot - x))$ with UV cutoff function φ. Then H can be defined as a self-adjoint operator on $L^2(R^d) \otimes F$ as $H = H_p \otimes 1 + H_i + 1 \otimes H_f$, where F denotes a boson Fock space and H_f the free field operator on F defined by the second quantization of dispersion relation $\omega(k) = \sqrt{|k|^2 + \nu^2}$. The heat semigroup generated by H, e^{-tH}, can be realized in terms of Brownian motion $(B_t)_{t \geqslant 0}$ on the Wiener space $(\mathcal{X}, \mathcal{F}, W^x)$ as

$$(F, e^{-tH}G) = \int_{R^d} dx E_W^x \left[e^{-\int_0^t V(B_s)ds} \left(F(B_0), J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s)ds)} J_t G(B_t) \right)_F \right].$$

In the case of massless: $\nu = 0$, we can show the boundedness of the integral kernel $J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s)ds)} J_t$ by the Baker-Campbell-Hausdorff formula. By this we discuss (0) the existence of ground state, (1) spatial decay of bound states of H, (2) super-exponential decay of the ground state of H.

[1] F. Hiroshima and O. Matte, Ground states and their associated Gibbs measures in the renormalized Nelson model, preprint 2018.

26. Vincel Hoang Ngoc Minh (University of Lille)

On the Drinfel'd series

Abstract: Starting from the equation KZ_3 and its differential Galois group, we describe a group of "associator", containing the unique Φ_{KZ} (determined by asymptotic conditions). We also exhibit non trivial examples of "associator" with rational coefficients.

[1] V.C. Bùi, G.H.E. Duchamp, V. Hoang Ngoc Minh; Structure of Polyzetas and Explicit Representation on Transcendence Bases of Shuffle and Stuffle Algebras, *J. of Sym. Comp.* 83 (2017), 93-111.

[2] G.H.E. Duchamp, V. Hoang Ngoc Minh, Q.H. Ngô; Harmonic sums and polylogarithms at negative multi-indices, *J. of Sym. Comp.* 83 (2017), 166-186.

27. Anna Jenčová (Mathematical Institute, Slovak Academy of Sciences, Slovakia)

Rényi relative entropies and noncommutative L_p-spaces

Abstract: The sandwiched version of quantum Rényi relative α-entropies was defined for density matrices and gained attention for its usefulness in quantum information theory. We propose an extension of these quantities to normal positive functionals on arbitrary von Neumann algebras, for the values $\alpha \in [\frac{1}{2}, 1) \cup (1, \infty]$. For this, we use the Kosaki interpolating family of noncommutative L_p-spaces with respect to a state. We show that these extensions coincide with the previously defined Araki-Masuda divergences defined by Berta et. al. [1]. We prove some of their properties, in particular the limit values for $\alpha \to 1$ and data processing inequality with respect to (completely) positive normal unital maps. This shows that the Araki relative entropy is monotone with respect to all positive normal unital maps. We also show that for any $\alpha \in (\frac{1}{2}, 1) \cup (1, \infty)$, equality in data processing inequality characterizes quantum channels that are sufficient with respect to a pair of states, in the sense introduced by Petz. For more details, see [2].

[1] M. Berta, V. B. Scholz, M. Tomamichel, Rényi divergences as weighted non-commutative vector valued L_p-spaces, arXiv:1608.05317 [math-ph].

[2] A. Jenčová, Rényi relative entropies and noncommutative L_p-spaces I and II, arXiv:1609.08462 and arXiv:1707.00047.

28. Artur Jeż (University of Wrocław)

Solving word equations in groups by recompression

Abstract: In this talk I will present an algorithm for solving equations in free groups, in particular, the algorithm gives a finite graph-like representation of all solutions. In this representation the edges are labelled with morphisms and the solutions are obtained as compositions of morphisms on the path from the designated starting vertex to the designated ending vertex. The main idea is to reduce the case of groups to semigroups (with regular constraints and involution) and then employ a simple technique of local recompression. The technique is based on local modification of variables (replacing X by aX or Xa) and iterative replacement of pairs of letters occurring in the equation by a 'fresh' letter, which can be seen as a bottom-up compression of the solution of the given equation. The crucial point of the analysis is that in this way we can keep the instance small, i.e. polynomial in the size of the input, thus guaranteeing termination of the whole procedure. I will also discuss generalisations of this algorithms to some other groups, which for instance include RAAGs.

29. Hong Chang Ji (Korea Advanced Institute of Science and Technology)

Central limit theorem for linear spectral statistics of deformed Wigner matrices

Abstract: In this talk, we consider large-dimensional Hermitian random matrices of the form $W = M + \vartheta V$ where M is a Wigner matrix and V is a random or deterministic, real, diagonal matrix whose entries are independent of M. The random matrix ensemble is known as *deformed Wigner matrices*. For analytic test function, we study the fluctuation of functional of the eigenvalues of W. We prove that the fluctuation can be decomposed into that of M and of V, and each of those weakly converges to a Gaussian distribution. The proof is based on the analysis of Stieltjes transform with self-comparison.

[1] H. C. Ji and J. O. Lee. Central limit theorem for linear spectral statistics of deformed Wigner matrices. arXiv:1712.00931.

[2] J. O. Lee and K. Schnelli. Local deformed semicircle law and complete delocalization for Wigner matrices with random potential. *J. Math. Phys.*, 54(10):103504, 62, 2013.

30. Tiju Cherian John (Indian Statistical Institute, Bangalore)

Infinite Mode Quantum Gaussian States

Abstract: Quantum Gaussian states on Bosonic Fock spaces are quantum versions of Gaussian distributions. A systematic study of the quantum Gaussian states in the infinite mode setting is initiated in this work. This naturally leads to Type I quasifree states on $CCR-$algebra and Hilbert-Schmidt, Trace class restrictions on the covariance operators. We characterize the quantum Gaussian states using the properties on covariance operators and extend many of the results of Parthasarathy [1] and [2] to the infinite mode case. This include various characterizations, convexity and symmetry properties. This is a joint work with B. V. Rajarama Bhat and R. Srinivasan.

[1] K. R. Parthasarathy, What is a Gaussian state?, *Commun. Stoch. Anal.* **4** (2010), no 2, 143-160..

[2] K. R. Parthasarathy, The symmetry group of Gaussian states in $L^2(\mathbb{R}^n)$, *Prokhorov and Contemporary Probability Theory*, vol. 33, Springer, Heidelberg, 2013, 349–369.

31. Paweł Józiak (Warsaw University of Technology)

On a quantum Bernstein Theorem

Abstract: The classical Theorem of Bernstein states that a random vector consisting of independent entries with the property that its entries are still independent after applying a generic rotation, is necessarily a Gaussian vector. A similar type of result was obtained by Nica, where independent was replaced with free, and Gauss law was replaced with Wigner law. We pursue a similar type of question with rotation replaced with quantum rotation. Staying in the framework of operator-valued free probability, we show that a random vector with free entries having the property that its entries remain free after applying a quantum family of rotations (described by a quotient of $\mathcal{O}(O_d^+)$) is necessarily a semicircular family of random variables, provided that this quantum family of rotations is not a subset of quantum hyperoctahedral group (the aforementioned quotient of $\mathcal{O}(O_d^+)$ does not factor through $\mathcal{O}(H_d^+)$). We also show that the result is optimal, in the sense that there exist non-semicircular free random variables that remain free after applying the rotations from H_d^+. Joint work with Kamil Szpojankowski.

32. Vladislav Kargin (Binghamton University, USA)

A 3D Ginibre point field

Abstract: We introduce a three-dimensional determinantal random point field using the concept of the quaternion determinant. Orthogonal polynomials on the space of pure quaternions are defined, and used to construct a kernel function similar to the Ginibre kernel. We find explicit formulas for the polynomials and the kernel, and calculate their asymptotics in the bulk and at the center of coordinates.

In more detail, let us define

$$\mathcal{K}_n(z,w) = (2\pi)^{-3/2} e^{-(|z|^2+|w|^2)/4} \sum_{k=0}^{n-1} \frac{P_k(z)\overline{P_k(w)}}{h_k}. \tag{1}$$

Here z and w are purely imaginary quaternions, which we identify with points in $\mathbb{R}^3$, and polynomials $P_k(z)$ are the monic quaternion polynomials of degree k, which are orthogonal with respect to the standard Gaussian measure on $\mathbb{R}^3$ and have norm $\sqrt{h_k}$. We prove that for pure

quaternions u, $P_n(us) = u^n Q_n(s)$, where $Q_n(s) \in \mathbb{Z}[s]$. In terms of these polynomials we derive a Christoffel-Darboux expression for the kernel.

Using kernel $\mathcal{K}_n(z,w)$ and the Moore-Dyson determinant for quaternion matrices, we define a random point field on $\mathbb{R}^3$ and show that (i) with high probability this field is contained in the ball of radius $2\sqrt{n}$; (ii) the average density of points in the ball is $1/\sqrt{n}$, and (iii) after rescaling the density has the form given by the function

$$\rho(s) := \frac{1}{(2\pi)^2}\frac{\sqrt{1-s^2}}{s^2} \text{ for } s \in (0,1). \tag{2}$$

We find that after suitable rescaling the correlation function for points σu and τv in the bulk of the point process is given by the formula

$$\frac{\sin(\tau-\sigma)}{\tau-\sigma}\frac{1-uv}{2}. \tag{3}$$

We also derive a scaling limit for the correlations at the center of coordinates.

33. Yasuyuki Kawahigashi (The University of Tokyo)

The relative Drinfeld commutant of a fusion category and α-induction

Abstract: We establish a correspondence among simple objects of the relative commutant of a full fusion subcategory in a larger fusion category in the sense of Drinfeld, irreducible half-braidings of objects in the larger fusion category with respect to the fusion subcategory, and minimal central projections in the relative tube algebra. Based on this, we explicitly compute certain relative Drinfeld commutants of fusion categories arising from α-induction for braided subfactors. We present examples arising from chiral conformal field theory.

34. Claus Köstler (University College Cork)

Central limit theorems for block characters of the infinite symmetric group

Abstract: Exchangeability of an infinite sequence of noncommutative random variables implies the existence of a central limit law for this sequence. So far little is known on the concrete form of the resulting central limit laws. Very recently we have identified these laws for the sequence of star generators of the infinite symmetric group with respect to its block characters. Given a block character, the emerging central limit law equals the distribution of a GUE random matrix, up to a convolution of the central limit law with a Gaussian distribution. My talk is based on joint work with Alexandru Nica.

35. Jakub Kośmider (Uniwersytet Jagielloński)

Unitary equivalence of weighted shifts

Abstract: Let $\mathcal{H}$ be a nonzero Hilbert space and $\boldsymbol{B}(\mathcal{H})$ be the algebra of bounded operators defined on $\mathcal{H}$. Let $\{S_n\}_{n\in\mathbb{Z}} \subseteq \boldsymbol{B}(\mathcal{H})$ be a two-sided sequence of bounded nonzero operators such that $\{||S_n||\}_{n\in\mathbb{Z}}$ is bounded. We say that an operator $S\colon \oplus_{n\in\mathbb{Z}}\mathcal{H} \to \oplus_{n\in\mathbb{Z}}\mathcal{H}$ is a *bilateral operator valued weighted shift* defined on $\mathcal{H}$ if for all $x \in \oplus_{n\in\mathbb{Z}}\mathcal{H}$ it holds that

$$Sx = (\dots, S_{-1}x_{-2}, \boxed{S_0 x_{-1}}, S_1 x_0, \dots),$$

where $x = (\dots, x_{-1}, \boxed{x_0}, x_1, \dots)$ and $\boxed{x_0}$ denotes the central element of x. The talk is based on my recent work regarding unitary equivalence of bilateral operator valued weighted shifts.

36. Jacek Krajczok (University of Warsaw)

Compact quantum groups with representations of bounded degree

Abstract: Compact quantum groups are objects which are analogues of compact Hausdorff groups in the realm of noncommutative geometry. One of their features is that coinverse, i.e. map which in the classical case is given by precomposition with an inverse, does not need to be bounded. Whenever coinverse is bounded, we say that compact quantum group $\mathbb{G}$ is of Kac type. I wish to present an argument which says that if every irreducible representation of a given compact quantum group $\mathbb{G}$ has dimension less that a fixed natural number, then $\mathbb{G}$ must be of Kac type. Proof is based on a certain inequality which involves quantum dimensions d_α and numbers related to operators $\rho_\alpha \in \mathrm{Mor}(U^\alpha, (U^\alpha)^{cc})$.

37. Bartosz Kosma Kwaśniewski (University of Białystok)

Nica-Toeplitz algebras associated with right tensor C^*-precategories over right LCM semigroups

Abstract: Tensor C^*-categories and all the more right-tensor C^*-categories, also called semi-tensor C^*-categories, arise naturally in quantum field theory and duality theory of compact (quantum) groups. Recently they played a fundamental role in a number of results with a flavor of geometric group theory. In this talk we show how these structures can be used to develop a theory of C^*-algebras modeled over semigroups. Nica-Teoplitz algebras are C^*-algebras associated to product systems - a general form of a semigroup action. While theory of semigroup C^*-algebras, developed by Li, is now well-established, C^*-algebras associated to product systems, through the crucial work of Fowler and Fowler-Raeburn, so far were only studied in the case of positive cones in quasi-lattice ordered groups. Our machinery substantially extends this theory in a number of ways and reveals some new phenomena:

1) We consider a larger class of semigroups that may contain invertible elements and need not be embeddable into a group.
2) We unify the theory of product systems over semigroups and Fell bundles over discrete groups.
3) We may study Doplicher-Roberts versions of Nica-Toeplitz algebras.
4) When the semigroup is not cancellative the canonical conditional expectation takes values outside the ambient algebra.
5) Geometric condition used by Fowler and Raeburn in their uniqueness theorems is in fact a condition that is necessary and sufficient for Doplicher-Roberts version of Nica-Toeplitz algebras.

The talk is based on a joint work with Nadia Larsen.

38. Seung-Hyeok Kye (Seoul National University)

Separable states with unique decomposition

Abstract: Entanglement is considered as one of the key resources in the current quantum information theory. Recall that a multi-partite state is said to be separable if it is a convex combination of pure product states. A non-separable state is called entangled. It is an important research topic to distinguish entanglement from separability, which is known to be NP-hard in general. We note that the boundary between separability and entanglement consists of faces of the convex set $\mathbb{S}$ of all separable states. In this context, it is important to understand the facial structures of the convex set $\mathbb{S}$. In this talk, we look for faces of $\mathbb{S}$ which are affinely isomorphic to a simplex, which is called a simplicial face. This is equivalent to search for separable states with

unique decomposition into the sum of pure product states. It is known that if a separable state has sufficiently small rank then it has a unique decomposition. We exhibit in this talk examples of separable states with unique decomposition whose ranks are full. More precisely, we construct one parameter families of faces isomorphic to the 9-dimensional simplex Δ_9 with ten extreme points, in the three qubit system. These also provide examples of three qubit separable states whose lengths are strictly bigger than the whole dimension.

39. Mi Ra Lee (Chungbuk National University)

One-Parameter Groups Involving Bogoluibov and Quantum Girsanov Transforms

Abstract: We study a Lie group of one-parameter transformations acting on white noise functionals associated with the Lie algebra generated by the scalar, annihilation operator, creation operator, conservation, generalized Gross Laplacian and its adjoint operator. The one-parameter groups are involving the Bogoliubov and quantum Girsanov transforms, Fourier-Gauss and Fourier-Mehler transforms and Weyl transform. We examine the unitarity conditions for the one-parameter groups. This talk is based on a joint work with Un Cig Ji.

40. Franz Lehner (TU Graz)

Quadratic and other forms in Free Probability

Abstract: We report on recent progress in our study of polynomials of low degree in free random variables. In the first part, continuing [1], we show that the following conditions are equivalent for a quadratic form in free random variables:

(a) it exhibits the phenomenon of cancellation of free cumulants

(b) it preserves infinite divisibility

(c) it can be written as a sum of commutators.

In addition, we present some central limit theorems whose limit laws exhibit interesting connections to classical analysis. In the second part we announce a method to compute the distribution of certain polynomials of degree three which are relevant for the free version of the Cantelli problem. This is joint work with W. Ejsmont (part 1), A. Piliszek, K. Szpojankowski and V. Vasilchuk (part 2).

[1] Wiktor Ejsmont and Franz Lehner, *Sample variance in free probability*, J. Funct. Anal. **273** (2017), no. 7, 2488–2520.

41. Romuald Lenczewski (Politechnika Wrocławska)

Random matrices, continuous circular systems and the triangular operator

Abstract: We present a Hilbert space approach based on direct integrals to the limit joint *-distributions of complex independent Gaussian random matrices with non-identical variances. For that purpose, we use a suitably defined family of creation and annihilation operators decomposed in terms of continuous circular systems of operators acting between the fibers of the considered direct integral of Hilbert spaces. In the case of square matrices with i.i.d. entries, we obtain the circular operators of Voiculescu, whereas in the case of upper-triangular matrices with i.i.d. entries, we obtain the triangular operators of Dykema and Haagerup. We apply this approach to give a bijective proof of a formula for *-moments of the triangular operator, using the enumeration formula of Chauve, Dulucq and Rechnizter for alternating ordered rooted trees.

42. Andrzej Łuczak (Faculty of Mathematics and Computer Science, Łódź University)

Entropy-preserving maps on von Neumann algebras

Abstract: Let $\mathfrak{M}$ be a semifinite von Neumann algebra with a normal semifinite faithful trace τ. For $h \in L^1(\mathfrak{M}, \tau)^+$, we define Segal's entropy, $H(h)$, by the formula

$$H(h) = \tau(h \log h) = \int_0^\infty t \log t \, \tau(e(dt)),$$

where h has the spectral representation $h = \int_0^\infty t \, e(dt)$. Let Φ be a normal positive unital linear map on $\mathfrak{M}$ such that $\tau \circ \Phi = \tau$. We are interested in characterising the equality $H(h) = H(\Phi(h))$ for $h \in L^1(\mathfrak{M}, \tau)^+$ with finite Segal's entropy. Two cases will be dealt with:

1. Φ represents a *repeatable measurement* which means that $\Phi = \Phi^2$, and h is an arbitrary element of $L^1(\mathfrak{M}, \tau)^+$;
2. Φ is arbitrary but h belongs to $\mathfrak{M} \cap L^1(\mathfrak{M}, \tau)^+$.

In particular, point 2 above applies to the von Neumann entropy in the algebra $\mathbb{B}(\mathcal{H})$ of all bounded linear operators on a Hilbert space.

43. Eugene Lytvynov (Swansea University)

An infinite dimensional umbral calculus – algebraic and analytic aspects

Abstract: The classical umbral calculus studies Sheffer polynomial sequences (including polynomial sequences of binomial type and Appell sequences) and related operators. In this talk, we will develop foundations of umbral calculus on the space $\mathcal{D}'$ of distributions on $\mathbb{R}^d$, which leads to a general theory of Sheffer polynomial sequences on $\mathcal{D}'$. We will construct a lifting of a Sheffer sequence on $\mathbb{R}$ to a Sheffer sequence on $\mathcal{D}'$. Examples of lifted polynomial sequences include the falling and rising factorials, Abel, Hermite, Charlier, and Laguerre polynomials on $\mathcal{D}'$. Some of these polynomials have already appeared in different branches of infinite dimensional (stochastic) analysis and played there a fundamental role. We will also study extensions of Sheffer operators (including umbral operators) to linear homeomorphisms on spaces of entire functions on $\mathcal{D}'_{\mathbb{C}}$, the complexification of $\mathcal{D}'$. Our results here extend the well known internal descriptions of the test spaces for Hida and Kondratiev distributions, respectively. The talk is based on joint papers with Dmitri Finkelshtein, Yuri Kondratiev, Maria João Oliveira, and Ludwig Streit.

44. Wojciech Matysiak (Politechnika Warszawska)

Multiplicity free actions and quantum Ornstein-Uhlenbeck processes

Abstract: We will consider some generalizations (related to group actions) of the binomial coefficients, defined for partitions of natural numbers. Next, we will use the generalized coefficients to define some simple birth and death processes on partitions, which have nice interpretations in terms of the dynamics of growth of a population with some distinguished subpopulations. Finally, we will show that the birth and death processes are essentially the quantum Ornstein-Uhlenbeck processes studied by Philippe Biane. This is joint work with Marcin Świeca (Politechnika Warszawska).

45. Wojciech Młotkowski (Uniwersytet Wrocławski)

Positive definite functions on Coxeter groups

Abstract: For an element w of a Coxeter group (W, S), with a reduced representation $w = s_{i_1} s_{s_2} \ldots s_{i_n}$ we define its *length* $|w| := n$, *colour* $S(w) := \{s_{i_1}, s_{i_2}, \ldots, s_{i_n}\}$ and *colour-length* $\|w\| := \#W(w)$. We are going to study positive definite functions on W which are constant on elements of the same length, colour and colour length respectively. The talk is based on a joint work with Światosław Gal and Marek Bożejko.

46. Nobuaki Obata (Graduate School of Information Sciences, Tohoku University)

Asymptotic Spectral Distributions for Strongly Regular Graphs and Bivariate Orthogonal Polynomials

Abstract: Let G be a strongly regular graph with eigenvalues $s(G) < r(G) \leqslant k(G)$, and $\bar{G}$ the complement which is known to be strongly regular. We consider the Cartesian powers G^n and $\bar{G}^n$, and their adjacency matrices A_n and $\bar{A}_n$, respectively. We are interested in the joint spectral distribution of $(A_n, \bar{A}_n)$ in the canonical tracial state. Following Hora [1], where the asymptotic spectral distributions of Hamming graphs $H(n, v) = K_v^n$ was first investigated, we obtain the asymptotic spectral distributions of $(A_n, \bar{A}_n)$ as $n \to \infty$ with proper scaling balance with $s(G), r(G), k(G)$. The limit distribution is described in terms of Gaussian and Poisson distributions, and the corresponding orthogonal polynomials are the limit of bivariate Krawtchouk polynomials, a particular class of Aomoto–Gelfand hypergeometric functions. It is our hope to establish along the above line a bivariate extension of the method of quantum decomposition [2, 4]. This talk is based on the recent joint work with J. Morales and H. Tanaka [3].

[1] A. Hora, Central limit theorems and asymptotic spectral analysis on large graphs, *Infin. Dimen. Anal. Quantum Probab. Relat. Top.* **1** (1998), 221–246.

[2] A. Hora and N. Obata: *Quantum Probability and Spectral Analysis of Graphs*, Springer, 2007.

[3] J. V. S. Morales, N. Obata and H. Tanaka, preprint, 2018.

[4] N. Obata, *Spectral Analysis of Growing Graphs*, Springer, 2017.

47. Przemysław Ohrysko (Chalmers University of Technology and Gothenburg University)

Spectral theory of Fourier-Stieltjes algebras

Abstract: In my talk I will discuss the recent developments on spectral properties of Fourier-Stieltjes algebras with a particular emphasis on the notion of naturality (equality with the closure of the image) and non-naturality of the spectrum of an element. It is an extensive project and due to time limitations most results will be outlined only. The motivation for research in this direction is the classical case of measures on locally compact Abelian groups for which it is well-known that surprising spectral behaviour occur. The prominent example is the Wiener-Pitt phenomenon: the spectrum of a measure may be much bigger than the closure of the image of it's Fourier-Stieltjes transform which implies the non-density of the dual group in the Gelfand space of the measure algebra. In our preprint we proved that Wiener-Pitt phenomenon is present for a wide-class of non-commutative groups in the setting of Fourier-Stieltjes algebras. Moreover, we extended the results of Hatori and Sato (on the possibility of writing any measure as a sum of two measures with a natural spectrum and a discrete measure) to discrete maximally almost periodic groups and the results of M. Zafran on the set of measures with natural spectrum and Fourier-Stieltjes transforms vanishing at infinity to arbitrary discrete groups. It should be noted that the assertions of some of the aforementioned facts can be proved via approach very similar to the original one but a lot of them require completely new ideas and there are also examples of facts that do not have non-commutative counterparts. Instead of measure theory we use a lot of operator theory to establish and use the notion of mutual singularity and absolute continuity of elements of Fourier-Stieltjes algebras. We also provide the spectral analysis of free Riesz products introduced by M. Bożejko. The talk is based on the preprint written in collaboration with Mateusz Wasilewski and available on arxiv.org with the identifier: 1705.05457.

48. Izumi Ojima (Research Origin for Dressed Photon, Yokohama)

Micro-Macro duality for Inductions/Reductions

Abstract: Paradoxical appearance of negative metrics in the processes of emergences will be analyzed from the viewpoint of Morse theory, induced representations and of imprimitivity systems.

49. Ostrovskyi Vasyl (Institute of Mathematics, NAS of Ukraine)

On decomposition of the identity operator into a linear combination of five projections

Abstract: In [1], the authors studied families of orthogonal projections $P_1, \ldots, P_n$ in a separable complex Hilbert space H, for which $P_1 + \cdots + P_n = \lambda I$, $\lambda > 0$. In particular, they described the set Σ_n of those $\lambda \in \mathbb{R}$, for which such a decomposition exists. While for $n \leqslant 4$, all such decompositions can be classified up to a unitary equivalence, for $n \geqslant 5$ such classification is possible only for λ in some discrete subset in Σ_n. Next problem which arised here is to study families of projections, for which

$$\alpha_1 P_1 + \cdots + \alpha_n P_n = I, \quad 0 < \alpha_j < 1, \quad n = 1, \ldots, n,$$

in particular, to study the structure of the set Ω_n of admissible weights $(\alpha_1, \ldots, \alpha_n) \subset (0,1)^n$. Again, for $n \leqslant 4$, both the structure of Ω_n and the unitary classification of all such decomposions have been obtained, while for $n \geqslant 5$ only partial results are known. Notice that such decompositions arise in frame theory as tight fusion frames. We discuss some recent results on the structure of sets Ω_n, $n \geqslant 5$, especially focusing our attention at the case $n = 5$. In particular, we show that if $\alpha_5 < \epsilon$ for small ϵ, then there exists $\Omega_4 \ni \beta = (\beta_1, \beta_2, \beta_3, \beta_4)$ with $\max_{1 \leqslant j \leqslant 4} |\alpha_j - \beta_j| < \epsilon$. This talk is based on joint results with Slavik Rabanovich.

[1] Kruglyak, S. A., Rabanovich, V. I., SamoÄlenko, Yu. S. On sums of projections. *Funct. Anal. Appl.* **36** (2002), no. 3, 182–195.

50. Lahcen Oussi (Uniwersytet Wrocławski)

Poisson type limit theorems for a noncommutative independence associated with positive symmetric cones

Abstract: We present Poisson type limit theorems for a noncommutative independence (the bm-independence), which is naturally associated with positive symmetric cones in euclidian spaces, including $\mathbb{R}_+^d$, the Lorentz cone in Minkowski spacetime and positive definite (real symmetric or complex hermitian) matrices. The geometry of the cones plays significant role in the study as well as the combinatorics of bm-ordered partitions.

51. Adam Paszkiewicz (Faculty of Mathematics and Computer Science, University of Lodz)

On sequences of products of contractions in B(H) and in any von Neumann algebra

Abstract: The following result will be presented. Let (x_m) be a sequence of vectors in a Hilbert space H (real or complex), satisfying 1) $\|x_1\| > \|x_2\| > \cdots$; 2) for a fixed m, $< x_m, x_n >= 0$ for almost all $n \in \mathbb{N}$. Then there exist orthogonal projections P, Q, R in H and a sequence $P_1, P_2, \dots \in \{P, Q, R\}$ such that (x_m) is a subsequence of some trajectory $(y_n, n \geqslant 0)$ given by $y_0 \in H$ and the condition $y_n = P_n y_{n-1}$ for $n \in \mathbb{N}$. We shall discuss a number of important results on sequences $(P_n \cdots P_1, n \in \mathbb{N})$ for $P_1, P_2, \dots \in \{Q_1, \ldots, Q_k\}$, with $Q_1, \ldots, Q_k$ being positive contractions on H. We give some ultimate solutions in the case $Q_1, \ldots, Q_k \in \mathcal{M}$ for some von Neumann algebra $\mathcal{M}$.

52. Karol A. Penson (Sorbonne Universités, Université Pierre et Marie Curie, Paris)

Integer ratios of factorials as Hausdorff moments

Abstract: Consider positive integers a, b with $\mathrm{nwd}(a,b) = 1$. The following three ratios of factorials for $n = 0, 1, \ldots$ turn out to be integers [1]:

$$u_1(a,b,n) = \frac{[(a+b)n]!}{(an)!(bn)!}\,,\ u_2(a,b,n) = \frac{(2an)!(bn)!}{(an)!(2bn)![(a-b)n]!} \quad (\text{for } a > b)\,,$$
$$\text{and} \quad u_3(a,b,n) = \frac{(2an)!(2bn)!}{(an)!(bn)![(a+b)n]!}\,.$$

The same applies to the fourth ratio in the form [2]

$$u_4(a,b,n) = \frac{[(2a+1)n]![(b+\frac{1}{2})n]!}{[(2b+1)n]![(a+\frac{1}{2})n]![(a-b)n]!} \quad (\text{for } a > b)\,.$$

We solve exactly the Hausdorff moment problem with moments given by $u_i(a,b,n)$ for $i = 1,\ldots,4$. We use the technique of the inverse Mellin transform and Meijer G functions to obtain the positive smooth measures $w_i(x)$ as well as their supports $(0, R_i(a,b))$. All these measures are U-shaped, are singular at the support bounds, and their singularities at $x = 0$ are of power-law type. The radii of convergence $r_i(a,b)$ of the ordinary generating functions (OGFs) of $u_i(a,b,n)$ satisfy $r_i(a,b) = [R_i(a,b)]^{-1}$ for $i = 1,\ldots,4$. All these OGFs are algebraic [1, 2, 3]. An attempt is made to understand to what extent the proven algebraicity of the OGFs is synchronized with the possible algebraicity of the $w_i(x)$. Joint work with G. H. E. Duchamp and G. Koshevoy, Univ. Paris XIII.

[1] Jonathan W. Bober, *Factorial ratios, hypergeometric series, and a family of step functions*, Journal of the London Mathematical Society **79** (2009), no. 2, 422–444.

[2] P. Bala, *"Some integer ratios of factorials"*, in the On-Line Encyclopedia of Integer Sequences (https://oeis.org/), sequence A276098.

[3] Fernando Rodriguez -Villegas, *Integral ratios of factorials and algebraic hypergeometric functions*, arXiv preprint math/0701362 (2007).

53. Paweł Pietrzycki (Jagiellonian University, Kraków)

Reduced commutativity of moduli of operators

Abstract: In 1973 M. R. Embry published a very influential paper studying the Halmos-Bram criterion for subnormality. In particular, she gave a characterization of the class of quasinormal operators in terms of powers of operators. Namely, bounded operator in Hilbert space is quasinormal if and only if the following condition holds

$$A^{*n}A^n = (A^*A)^n \qquad \text{for all} \quad n \in \mathbb{N}. \tag{4}$$

This leads to the following question: is it necessary to assume that the equality in (4) holds for all $n \in \mathbb{N}$? To be more precise we ask for which subset $S \subset \mathbb{N}$ the following system of operator equations:

$$A^{*s}A^s = (A^*A)^s \qquad \text{for all} \quad s \in S \tag{5}$$

implies the quasinormality of A. We will prove that operator A is quasinormal if and only if it satisfies the system of equations (5) with $S = \{p, m, m+p, n, n+p\}$. This theorem generalizes Embry's characterization of quasinormality of bounded operators. We obtain a new characterization of the normal operators which resembles that for the quasinoraml operators.

[1] P. Pietrzycki, The single equality $A^{*n}A^n = (A^*A)^n$ does not imply the quasinormality of weighted shifts on rootless directed trees, *J. Math. Anal. Appl* 435 (2016), 338-348.

[2] P. Pietrzycki, Reduced commutativity of moduli of operators, arXiv:1802.01007, 2018.

54. Hanna Podsędkowska (Faculty of Mathematics and Computer Sciences, University of Łódź)

General Quantum Systems-Strong subadditivity of entropy

Abstract: We show that for Segal entropy defined for states on an arbitrary von Neumann algebra with normal faithful semifinite trace strong subadditivity holds. We present also some other related properties of this generalized entropy, in particular, the concavity of $S(\rho_{12})-S(\rho_2)$, the triangle inequality, and a generalization of Araki-Lieb inequality.

The strong subadditivity hypothesis for entropy, which is the main subject of this presentation, has a long history. It was first conjectured by O.E. Lanford and D.W. Robinson. A proof of this hypothesis was given in 1973 by E.H. Lieb and M.B. Ruskai as a consequence of the celebrated Lieb inequallity. The theorem was stated (and proved) for the case of the full algebra $\mathbb{B}(\mathcal{H})$ of all bounded linear operators on a Hilbert space and the canonical trace. Later various generalisations of this theorem, as well as related results, turned up. In particular, M.B. Ruskai extended the triangle inequality to the case of finite von Neumann algebras and bounded density matrices. So far, the strong subadditivity of entropy for semifinite algebras, even for bounded density matrices, has remained unproven, although it was stated as a conjecture by Ruskai. We present the strong subadditivity theorem for the Segal entropy, in a pretty general case of semifinite von Neumann algebra and arbitrary states. Some other properties of entropy mentioned above, are given here as consequences of the strong subadditivity theorem.

Since the strong subadditivity of entropy is fundamental in quantum information theory not only on account of its physical applications but also because of mathematical consequences, it seems important and interesting to have this property for the general case of Segal's entropy in semifinite von Neumann algebras.

55. Sutanu Roy (NISER Bhubaneswar, India)

Quantum symmetries of the twisted tensor products of C*-algebras

Abstract: We consider the construction of twisted tensor products in the category of C*-algebras equipped with orthogonal filtrations and under certain assumptions on the form of the twist compute the corresponding quantum symmetry group, which turns out to be the generalised Drinfeld double of the quantum symmetry groups of the original filtrations. We show how these results apply to a wide class of crossed products of C*-algebras by actions of discrete groups. We also discuss an example where the hypothesis of our main theorem is not satisfied and the quantum symmetry group is not a generalised Drinfeld double. This is a joint work with Jyotishman Bhowmick, Arnab Mandal and Adam Skalski.

56. Konrad Schmüdgen (Universität Leipzig)

Transition Probability of States on $*$-Algebras

Abstract: Let f and g be states on a unital complex $*$-algebra A. The transition probability $P_A(f,g)$ is defined as the supremum of $|\langle\varphi_f,\varphi_g\rangle|^2$ over all $*$-representations π of A for which f and g are represented as vector states with corresponding vectors φ_f and φ_g. This number $d(f,g) = 2-\sqrt{P_A(f,g)}$ is the Bures distance of f and g. We present some results about the number $P_A(f,g)$ dealing with unbounded representations. The main technical ingredient is a description of $P_A(f,g)$ in terms of intertwiners related to the GNS representations of f and g. Under some regularity assumptions we prove: If A is the Weyl algebra and $f(a) = \mathrm{Tr}\ ta$

and $g(a) = \text{Tr } sa$, then $P_A(f,g) = (\text{Tr } |t^{1/2}s^{1/2}|)^2$. If A is an algebra of functions on a locally compact space X and f and g are given by densities η and ξ of a measure μ on X, then $P_A(f,g) = (\int_X \sqrt{\eta(x)\xi(x)}\, d\mu(x))^2$.

57. Katarzyna Siudzińska (Nicolaus Copernicus University)

Quantum maps covariant with respect to the group generated by the Weyl operators

Abstract: We construct the linear maps that are covariant with respect to the unitary representations of the finite groups. In particular, we focus on the groups generated by the unitary operator bases; that is, the Weyl operators and their tensor products. Necessary conditions are given for the complete positivity of these maps. Next, we follow the method of constructing the irreducibly covariant quantum channels (completely positive, trace-preserving maps). It turns out that our choices of finite groups lead to the Weyl channels and multipartite Weyl channels. Interestingly, these channels have real eigenvalues if and only if they are also covariant with respect to the transformation matrices that connect the equivalent representations of the group. Introducing even more symmetry constraints results in the generalized Pauli channels, whose construction involves the notion of maximal sets of commuting operators. Finally, we present several examples of the positive but not completely positive irreducibly covariant maps. As their physical application, we use these maps to determine whether or not the open quantum system admits the Markovian evolution.

58. Paul Skoufranis (York University)

Bi-Free Versions of Entropy

Abstract: In a series of papers, Voiculescu generalized the notions of entropy and Fisher information to the free probability setting. In particular, the notions of free entropy have several applications in the theory of von Neumann algebras and free probability such as demonstrating certain von Neumann algebras do not have property Gamma, demonstrating the absence of atoms in the distributions of polynomials of random matrices, and the construction of free monotone transport. With the recent bi-free extension of free probability being sufficiently developed, it is natural to ask whether there are bi-free extensions of Voiculescu's notions of free entropy. In this talk, we will provide an introduction to the notions of free entropy, discuss bi-free versions of entropy, and discuss the difficulties and peculiarities that occur in bi-free entropy theory. This is joint work with Ian Charlesworth.

59. Kamil Szpojankowski (Warsaw University of Technology)

A relation between c-freeness and infinitesimal freeness

Abstract: I will consider two extensions of free probability: c-freeness and infinitesimal freeness. In both frameworks one adds to a standard non-commutative probability space $(\mathcal{A}, \varphi)$ an additional functional. In the case of c-feeness one adds χ such that $\chi(1_{\mathcal{A}}) = 1$, in the case of infinitesimal freeness one considers φ' with the property $\varphi'(1_{\mathcal{A}}) = 0$. I will recall both notions of independence and combinatorial tools which characterize them. In the c-free case I will show a connections between cumulants defined by Bożejko, Leinert and Speicher and cumulants studied by Cabanal-Duvillard. I will recall also non-crossing partitions of type B which appear in the context of infinitesimal freeness. The main result which I will present is a construction, which uses the functional χ in order to define φ' in such a way, that structures which are c-free with respect to (φ, χ) become infinitesimally free with respect to (φ, φ'). I will also discuss some difficulties related with this construction. Based on a joint work with M. Fevrier (Paris, France), M. Mastnak (Halifax, Canada) and A. Nica (Waterloo, Canada).

60. Wojciech Tarnowski (Jagiellonian University in Kraków)

Eigenvector non-orthogonality in non-Hermitian random matrices

Abstract: Complexity of eigenvalues of non-Hermitian random matrices challenges standard techniques from free probability. Besides that, non-Hermiticity often leads to non-orthogonality of eigenvectors, which calls for new tools to study such properties. I present a formalism for the calculation of the correlation functions, which are the local averages of the matrix of overlaps between left and right eigenvectors. It relies on the quaternionic resolvent and a Kronecker product of two quaternionic resolvent for the two-point function, generalizing nicely concepts from standard free probability. Moment expansion in the large N limit leads to the Bethe-Salpeter equations, which can be solved in full generality from the knowledge of cumulants. A very simple and general formula can be obtained for the class of biunitarily invariant ensembles, which in the large N limit are R-diagonal operators. This result generalizes the Haagerup-Larsen theorem. Based on:

[1] S. Belinschi, M. A. Nowak, R. Speicher, W. Tarnowski *Squared eigenvalue condition numbers and eigenvector correlations from the single ring theorem* J. Phys. A: Math. Th. **50**, 105204 (2017).

[2] M. A. Nowak, W. Tarnowski *Probing non-orthogonality of eigenvectors in non-Hermitian matrix models: diagrammatic approach.* Preprint arXiv:1801.02526 (2018).

61. Mariusz Tobolski (IMPAN)

Local-triviality dimension of actions of compact quantum groups

Abstract: We introduce the local-triviality dimension of an action of a compact quantum group on a unital C*-algebra using completely positive contractive order zero maps of Winter and Zacharias. In the case of a compact Hausdorff group acting on a compact Hausdorff space our definition recovers the usual local triviality of a compact principal bundle. Actions with finite local-triviality dimension are automatically free and there exists an analog of an n-universal space (in the sense of Steenrod) for any compact quantum group $\mathbb{G}$. Our main motivating examples are the Matsumoto-Hopf fibration and antipodal actions of free orthogonal quantum spheres. As the main application, we prove a new Borsuk-Ulam-type conjecture of Baum, Dąbrowski and Hajac in the case where the compact quantum group $\mathbb{G}$ admits a classical subgroup whose induced action has finite local-triviality dimension.

62. Lyudmila Turowska (Chalmers University of Technology and University of Gothenburg)

Shilov boundary for "holomorphic functions" on a quantum matrix ball

Abstract: The Shilov boundary of a compact Hausdorff space X relative to a uniform algebra $\mathcal{A}$ in $C(X)$ is the smallest closed subset $K \subset X$ such that every function in $\mathcal{A}$ achieves its maximum modulus on K. This notion is encountered, in particular, in the theory of analytic functions in relation to the maximum modules principle. We will be interested in its non-commutative analog. The latter was introduced by W. Arveson.

In the middle of 90s, within the framework of the quantum group theory, L.Vaksman and his coauthors started a "quantisation" of bounded symmetric domains. One of the simplest of such domains is the matrix ball $\mathbb{D} = \{z \in Mat_m : zz^* < I\}$, where Mat_m is the algebra of complex $m \times m$ matrices. The Shilov boundary of $\mathbb{D}$ relative to the algebra of holomorphic functions in $C(\overline{\mathbb{D}})$ is the set of unitary $m \times m$-matrices. In this talk I will discuss the Shilov boundary ideal for the q-analog of holomorphic functions on the unit ball. This is a joint work with O.Bershtein and O.Gisselson.

63. Mateusz Wasilewski (Institute of Mathematics, Polish Academy of Sciences)

Approximation properties of q-Araki-Woods algebras

Abstract: I will report on recent results about type III counterparts of q-Gaussian algebras – the q-Araki-Woods algebras. In the first part of the talk I will introduce the main tools: the Wick formula and the second quantisation. In the second half I plan to discuss the Haagerup property and complete metric approximation property, and outline the proofs. This is joint work with Stephen Avsec and Michael Brannan.

64. Rafał Wieczorek (Łódź University)

Optimal measurement in the quantum statistical decision theory

Abstract: Let $\rho_1, \rho_2, \ldots$ be normal states on a von Neumann algebra $\mathfrak{M}$ which can occur with some a priori probabilities $\pi = (\pi_1, \pi_2, \ldots)$. We want to find a measurement (POVM– positive operator valued measure) $M = (M_1, M_2, \ldots), M_i \in \mathfrak{M}$ which minimizes Bayes risk i.e.

$$r(M, \pi) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \pi_i L(i, j) \rho_i(M_j),$$

where L is a loss function. We want to present some estimates of Bayes risk (e.g. Holevo–Curlander bounds, entropic bound [2]) for arbitrary von Neumann algebra $\mathfrak{M}$ and generalization of Holevo asymptotic theorem [1] for infinitely dimensional Hilbert space.

[1] Holevo A. S., *On Asymptotically Optimal Hypothesis Testing in Quantum Statistics*, Theory of Probability and its Applications **23**(2) (1978), 411–415.

[2] Wieczorek R., Podsędkowska H., *Entropic upper bound for Bayes risk in the quantum case*, Probability and Mathematical Statistics.

65. Hyun Jae Yoo (Hankyong National University)

Quantum Markov chains of open quantum random walks

Abstract: In this talk we discuss the quantum Markov chains initiated by Accardi. It was developed to extend the classical Markov chains to noncommutative spaces. Here we will specifically consider the open quantum random walks as quantum Markov chains. Then we discuss the reducibility and irreducibility of the open quantum random walks based on the theory of quantum Markov chains. Some examples will be provided. In particular, the classical Markov chains will be considered as open quantum random walks, and hence as quantum Markov chains. This talk is based on the joint work with Ameur Dhahri and Chul Ki Ko.

66. Joachim Zacharias (University of Glasgow)

Noncommutative dimension and almost finiteness

Abstract: In recent years noncommutative dimension concepts such as nuclear dimension have proved to be very important in the classification of simple nuclear C*-algebras. Finiteness of nuclear dimension has turned out to be the correct regularity condition to make the classification work. The other important regularity condition, now known to be equivalent to finite nuclear dimension is Z-stability. One of the main problems in classification is now to prove that certain given algebras are actually classifiable e.g. have finite nuclear dimension or are Z-stabie. Some years ago Hiroki Matui introduced almost finiteness as a regularity condition for groupoids. Almost finiteness has turned out to be a very powerful tool to prove Z-stability for crossed product algebras coming from discrete amenable groups acting minimally on compact metric spaces i.e. on abelian C*-algebras. We extend the definition of almost finiteness to the setting of groups acting on noncommutative coefficient algebras. The resulting concept combines aspects of the definition of nuclear dimension and the abelian case. We obtain Z-stablity for almost finite actions. This is the joint with Joan Bosa, Francesc Perera and Jianchao Wu.