Particle-hole duality in the continuum and determinantal point processes

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Abstract

Abstract: Let X be an underlying locally compact Polish space equipped with a Borel measure σ. Let $K(x,y) : X^2 \to \mathbb{C}$ and let K denote the integral operator in $L^2(X,\sigma)$ with integral kernel $K(x,y)$. A point process μ on X is called determinantal with the correlation operator K if the correlation functions of μ are given by $k^{(n)}(x_1,\dots,x_n) = \det[K(x_i,x_j)]_{i,j=1,\dots,n}$. If the operator K is self-adjoint, a determinantal point process with correlation operator K exists if and only if K is locally trace-class and $0 \le K \le 1$. Each determinantal point process with a Hermitian correlation kernel can be understood as the (spectral measure of) the particle density $\rho(x) = \partial_x^\dagger \partial_x$ ($x \in X$), where the operator-valued distributions $\partial_x^\dagger$, ∂_x come from a gauge-invariant quasi-free representation of the canonical anticommutation relations (CAR). If the space X is discrete and divided into two disjoint parts, X_1 and X_2, by exchanging particles and holes on the X_2 part of the space, one obtains from a determinantal point process with correlation kernel K a determinantal point process with correlation kernel $\widehat{K} = KP_1 + (1-K)P_2$, where P_i is the orthogonal projection onto $L^2(X_i,\sigma)$. In particular, the operator $\widehat{K}$ is J-self-adjoint. In the case where the space X is continous, a direct procedure of swapping particles and holes makes no sense. Nevertheless, we prove that it is possible to carry out such a procedure by swapping creation operators $\partial_x^\dagger$ with annihilation operators ∂_x on the X_2 part of the space. This leads to a quasi-free representation of CAR and the corresponding particle density is a determinantal point process with correlation operator $\widehat{K}$, which is J-self-adjoint.