

Structure of bicentralizer algebras and inclusions of type III factors

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Abstract

Let M be a type III₁ factor with a faithful normal state φ . The *bicentralizer* of (M, φ) , denoted by $B(M, \varphi)$ is the von Neumann subalgebra of M consisting of all $a \in M$ for which $\lim_n \|ax_n - x_na\|_\varphi = 0$ holds for every bounded sequence $(x_n)_{n=1}^\infty$ in M which satisfies $\lim_n \|x_n\varphi - \varphi x_n\| = 0$. It was introduced by Connes in his strategy to prove the uniqueness of the injective type III₁ factor. Namely he showed that an injective type III₁ factor with the trivial bicentralizer is isomorphic to the Araki–Woods factor, which was affirmatively solved by Haagerup. Whether every type III₁ factor with separable predual has trivial bicentralizer is an open problem. Thanks to Haagerup’s characterization of type III₁ factors with trivial bicentralizers, the bicentralizer problem is still of importance in the structure theory of type III₁ factors. Connes also showed that the bicentralizers are independent of φ up to a canonical isomorphism. Recently Haagerup observed that the idea of Connes’ isomorphism between bicentralizers can be enhanced to construct a canonical flow β^φ on $B(M, \varphi)$ which has interesting properties. Later, the flow was independently discovered by Marrakchi. We generalize the idea to the relative setting to investigate the structure of the relative bicentralizer algebra (defined by Masuda) $B(N \subset M, \varphi)$ for inclusions of von Neumann algebras with normal expectation where N is a type III₁ subfactor and $\varphi \in N_*$ is a faithful state. We construct a canonical flow $\beta^\varphi : \mathbb{R}_+^* \curvearrowright B(N \subset M, \varphi)$ on the relative bicentralizer algebra and we show that the W^* -dynamical system $(B(N \subset M, \varphi), \beta^\varphi)$ is independent of the choice of φ up to a canonical isomorphism. We then discuss the relationship between the properties of the flow and the structure of the inclusion $N \subset M$.

This is joint work with Uffe Haagerup, Cyril Houdayer and Amine Marrakchi.