Harmonic factorization of matrix'polynomials and Imanent'computations

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18 Workshop, Będlewo 2018

Abstract

Let $g = gl_n(\mathbb{C})$ be the Lie algebra of all n by n matrices over the complex numbers and $G = GL_n(\mathbb{C})$ be the general linear group over the complex numbers. G acts on g by conjugation : $a \circ g = g^a = aga^{-1}$. Let $S(g)$ denote the ring of polynomials whose indeterminates are the elements of the matrices of g. The $\circ$ action of G on g induces an action on $S(g)$ we denote also by $\circ$.

Let $J_G(g) \subseteq S(g)$ be the ring of G-invariant polynomials and $H \subseteq S(g)$ (H for harmonic) is the subspace of the polynomials vanishing under all differential operators coming from G invariants. By Kostant(JAMS1963) any polynomial in S(g) can be written uniquely as linear combination of harmonics with G invariants as coefficients. This is called the *harmonic factorization*.

For given $f \in S(g)$ and a given diagonal matrix t, define a function $\widetilde{f}^t$ from G to the complex numbers by $\widetilde{f}^t(a) = f(a \circ t)$. The λ immanent of a matrix A defined by $Im_\lambda(A) = \sum_{\sigma \in S_n} \chi_\lambda(\sigma) \prod_i A_{i\sigma(i)}$ Where χ_λ is the S_n irreducible character of $\lambda \vdash n$. For some family of partitions λ we give the full harmonic factorization of Im_λ. By using the normality of the G-orbits of t non degenerate diagonal matrices t, and by details from Kostant (JAMS1963)and Matsuzawa (comm in algebra 1998) we show that the dimension of the space spanned by $\widetilde{Im^t_\lambda}$ where t runs over all the diagonal matrices, is closely related to the function $\sum_\eta S^2_\lambda(x^\eta)|_{x=1,1,1,..}$, summed over $\eta \vdash n$, with x specialized to 1 in each coordinate and S_λ being the Schur function of the partition λ

Some computational aspects of this algebraic results are considered.