# Integral kernels of semigroup generated by a model in quantum field theory


Fumio Hiroshima

Faculty of Mathematics, Kyushu University
hiroshima@math.kyushu-u.ac.jp



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## Abstract

This is the joint work with O.Matte [1]. The Nelson model in scalar quantum field theory is defined by Schrödinger operator $H_p = -\Delta/2 + V$ in $L^2(R^d_x)$ linearly coupled to a scalar bose field $H_i = \phi(\varphi(\cdot - x))$ with UV cutoff function $\varphi$. Then $H$ can be defined as a self-adjoint operator on $L^2(R^d) \otimes F$ as

$$H = H_p \otimes 1 + H_i + 1 \otimes H_f,$$

where $F$ denotes a boson Fock space and $H_f$ the free field operator on $F$ defined by the second quantization of dispersion relation $\omega(k) = \sqrt{|k|^2 + \nu^2}$. The heat semigroup generated by $H$, $e^{-tH}$, can be realized in terms of Brownian motion $(B_t)_{t\geq 0}$ on the Wiener space $(\mathcal{X}, \mathcal{F}, W^x)$ as

$$(F, e^{-tH}G) = \int_{R^d} dx E^x_W \left[ e^{-\int_0^t V(B_s)ds} \left( F(B_0), J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s)ds)} J_t G(B_t) \right)_F \right].$$

In the case of massless : $\nu = 0$, we can show the boundedness of the integral kernel $J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s)ds)} J_t$ by the Baker-Campbell-Hausdorff formula. By this we discuss (0) the existence of ground state, (1) spatial decay of bound states of $H$, (2) super-exponential decay of the ground state of $H$.



[1] F.Hiroshima and O.Matte, Ground states and their associated Gibbs measures in the renormalized Nelson model, preprint 2018.