Rényi relative entropies and noncommutative L_p-spaces

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Abstract

The sandwiched version of quantum Rényi relative α-entropies was defined for density matrices and gained attention for its usefulness in quantum information theory. We propose an extension of these quantities to normal positive functionals on arbitrary von Neumann algebras, for the values $\alpha \in [\frac{1}{2}, 1) \cup (1, \infty]$. For this, we use the Kosaki interpolating family of noncommutative L_p-spaces with respect to a state. We show that these extensions coincide with the previously defined Araki-Masuda divergences defined by Berta et. al. [1]. We prove some of their properties, in particular the limit values for $\alpha \to 1$ and data processing inequality with respect to (completely) positive normal unital maps. This shows that the Araki relative entropy is monotone with respect to all positive normal unital maps. We also show that for any $\alpha \in (\frac{1}{2}, 1) \cup (1, \infty)$, equality in data processing inequality characterizes quantum channels that are sufficient with respect to a pair of states, in the sense introduced by Petz. For more details, see [2].

[1] M. Berta, V. B. Scholz, M. Tomamichel, Rényi divergences as weighted non-commutative vector valued L_p-spaces, arXiv:1608.05317 [math-ph]

[2] A. Jenčová, Rényi relative entropies and noncommutative L_p-spaces I and II, arXiv:1609.08462 and arXiv:1707.00047