Nica-Toeplitz algebras associated with right tensor $C^*$-precategories over right LCM semigroups

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18 Workshop, Będlewo 2018

Abstract

Tensor $C^*$-categories and all the more right-tensor $C^*$-categories, also called semitensor $C^*$-categories, arise naturally in quantum field theory and duality theory of compact (quantum) groups. Recently they played a fundamental role in a number of results with a flavor of geometric group theory. In this talk we show how these structures can be used to develop a theory of $C^*$-algebras modeled over semigroups.

Nica-Toeplitz algebras are $C^*$-algebras associated to product systems - a general form of a semigroup action. While theory of semigroup $C^*$-algebras, developed by Li, is now well-established, $C^*$-algebras associated to product systems, through the crucial work of Fowler and Fowler-Raeburn, so far were only studied in the case of positive cones in quasi-lattice ordered groups. Our machinery substantially extends this theory in a number of ways and reveals some new phenomena:

1) We consider a larger class of semigroups that may contain invertible elements and need not be embeddable into a group.

2) We unify the theory of product systems over semigroups and Fell bundles over discrete groups.

3) We may study Doplicher-Roberts versions of Nica-Toeplitz algebras.

4) When the semigroup is not cancellative the canonical conditional expectation takes values outside the ambient algebra.

5) Geometric condition used by Fowler and Raeburn in their uniqueness theorems is in fact a condition that is necessary and sufficient for Doplicher-Roberts version of Nica-Toeplitz algebras.

The talk is based on a joint work with Nadia Larsen.