Entropy-preserving maps on von Neumann algebras

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Abstract

Let $\mathfrak{M}$ be a semifinite von Neumann algebra with a normal semifinite faithful trace τ.

For $h \in L^1(\mathfrak{M}, \tau)^+$, we define Segal's entropy, $H(h)$, by the formula

$$H(h) = \tau(h \log h) = \int_0^\infty t \log t \, \tau(e(dt)),$$

where h has the spectral representation

$$h = \int_0^\infty t \, e(dt).$$

Let Φ be a normal positive unital linear map on $\mathfrak{M}$ such that

$$\tau \circ \Phi = \tau.$$

We are interested in characterising the equality

$$H(h) = H(\Phi(h))$$

for $h \in L^1(\mathfrak{M}, \tau)^+$ with finite Segal's entropy. Two cases will be dealt with:

1. Φ represents a *repeatable measurement* which means that

$$\Phi = \Phi^2,$$

and h is an arbitrary element of $L^1(\mathfrak{M}, \tau)^+$;

2. Φ is arbitrary but h belongs to $\mathfrak{M} \cap L^1(\mathfrak{M}, \tau)^+$.

In particular, point 2 above applies to the von Neumann entropy in the algebra $\mathbb{B}(\mathcal{H})$ of all bounded linear operators on a Hilbert space.