An infinite dimensional umbral calculus – algebraic and analytic aspects

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Abstract

The classical umbral calculus studies Sheffer polynomial sequences (including polynomial sequences of binomial type and Appell sequences) and related operators. In this talk, we will develop foundations of umbral calculus on the space $\mathcal{D}'$ of distributions on $\mathbb{R}^d$, which leads to a general theory of Sheffer polynomial sequences on $\mathcal{D}'$. sequence of monic polynomials on $\mathcal{D}'$, a polynomial sequence of binomial type, and a Sheffer sequence. We present equivalent conditions for a sequence of monic polynomials on $\mathcal{D}'$ to be of binomial type or a Sheffer sequence, respectively. We will construct a lifting of a Sheffer sequence on $\mathbb{R}$ to a Sheffer sequence on $\mathcal{D}'$. Examples of lifted polynomial sequences include the falling and rising factorials, Abel, Hermite, Charlier, and Laguerre polynomials on $\mathcal{D}'$. Some of these polynomials have already appeared in different branches of infinite dimensional (stochastic) analysis and played there a fundamental role. We will also study extensions of Sheffer operators (including umbral operators) to linear homeomorphisms on spaces of entire functions on $\mathcal{D}'_{\mathbb{C}}$, the complexification of $\mathcal{D}'$. Our results here extend the well known internal descriptions of the test spaces for Hida and Kondratiev distributions, respectively. The talk is based on joint papers with Dmitri Finkelshtein, Yuri Kondratiev, Maria João Oliveira, and Ludwig Streit.