Spectral theory of Fourier-Stieltjes algebras

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Abstract

In my talk I will discuss the recent developments on spectral properties of Fourier-Stieltjes algebras with a particular emphasis on the notion of naturality (equality with the closure of the image) and non-naturality of the spectrum of an element. It is an extensive project and due to time limitations most results will be outlined only. The motivation for research in this direction is the classical case of measures on locally compact Abelian groups for which it is well-known that surprising spectral behaviour occur. The prominent example is the Wiener-Pitt phenomenon: the spectrum of a measure may be much bigger than the closure of the image of it's Fourier-Stieltjes transform which implies the non-density of the dual group in the Gelfand space of the measure algebra. In our preprint we proved that Wiener-Pitt phenomenon is present for a wide-class of non-commutative groups in the setting of Fourier-Stieltjes algebras. Moreover, we extended the results of Hatori and Sato (on the possibility of writing any measure as a sum of two measures with a natural spectrum and a discrete measure) to discrete maximally almost periodic groups and the results of M. Zafran on the set of measures with natural spectrum and Fourier-Stieltjes transforms vanishing at infinity to arbitrary discrete groups. It should be noted that the assertions of some of the aforementioned facts can be proved via approach very similar to the original one but a lot of them require completely new ideas and there are also examples of facts that do not have non-commutative counterparts. Instead of measure theory we use a lot of operator theory to establish and use the notion of mutual singularity and absolute continuity of elements of Fourier-Stieltjes algebras. We also provide the spectral analysis of free Riesz products introduced by M. Boejko. The talk is based on the preprint written in collaboration with Mateusz Wasilewski and available on arxiv.org with the identifier: 1705.05457.