

On sequences of products of contractions in $B(H)$ and in any von Neumann algebra.

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Abstract

The following result will be presented. Let (x_m) be a sequence of vectors in a Hilbert space H (real or complex), satisfying 1) $\|x_1\| > \|x_2\| > \dots$; 2) for a fixed m , $\langle x_m, x_n \rangle = 0$ for almost all $n \in \mathbb{N}$. Then there exist orthogonal projections P, Q, R in H and a sequence $P_1, P_2, \dots \in \{P, Q, R\}$ such that (x_m) is a subsequence of some trajectory $(y_n, n \geq 0)$ given by $y_0 \in H$ and the condition $y_n = P_n y_{n-1}$ for $n \in \mathbb{N}$.

We shall discuss a number of important results on sequences $(P_n \cdots P_1, n \in \mathbb{N})$ for $P_1, P_2, \dots \in \{Q_1, \dots, Q_k\}$, with Q_1, \dots, Q_k being positive contractions on H . We give some ultimate solutions in the case $Q_1, \dots, Q_k \in \mathcal{M}$ for some von Neumann algebra \mathcal{M} .