Reduced commutativity of moduli of operators

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Abstract

In 1973 M. R. Embry published a very influential paper studying the Halmos-Bram criterion for subnormality. In particular, she gave a characterization of the class of quasinormal operators in terms of powers of operators. Namely, bounded operator in Hilbert space is quasinormal if and only if the following condition holds

$$A^{*n}A^n = (A^*A)^n \qquad \text{for all} \quad n \in \mathbb{N}. \tag{1}$$

This leads to the following question: is it necessary to assume that the equality in (1) holds for all $n \in \mathbb{N}$? To be more precise we ask for which subset $S \subset \mathbb{N}$ the following system of operator equations:

$$A^{*s}A^s = (A^*A)^s \qquad \text{for all} \quad s \in S \tag{2}$$

implies the quasinormality of A.

We will prove that operator A is quasinormal if and only if it satisfies the system of equations (2) with $S = \{p, m, m+p, n, n+p\}$. This theorem generalizes Embry's characterization of quasinormality of bounded operators. We obtain a new characterization of the normal operators which resembles that for the quasinoraml operators.

[1] P. Pietrzycki, The single equality $A^{*n}A^n = (A^*A)^n$ does not imply the quasinormality of weighted shifts on rootless directed trees, *J. Math. Anal. Appl* 435, 338-348 (2016)

[2] P. Pietrzycki, Reduced commutativity of moduli of operators, arXiv preprint arXiv:1802.01007, 2018.