General Quantum Systems-Strong subadditivity of entropy

Hanna Podsędkowska

Faculty of Mathematics and Computer Sciences, University of Łódź
hpodsedk@math.uni.lodz.pl

18 Workshop, Będlewo 2018

Abstract

We show that for Segal entropy defined for states on an arbitrary von Neumann algebra with normal faithful semifinite trace strong subadditivity holds. We present also some other related properties of this generalized entropy, in particular, the concavity of $S(\rho_{12}) - S(\rho_2)$, the triangle inequality, and a generalization of Araki-Lieb inequality.

The strong subadditivity hypothesis for entropy, which is the main subject of this presentation, has a long history. It was first conjectured by O.E. Lanford and D.W. Robinson. A proof of this hypothesis was given in 1973 by E.H. Lieb and M.B. Ruskai as a consequence of the celebrated Lieb inequality. The theorem was stated (and proved) for the case of the full algebra $\mathbb{B}(\mathcal{H})$ of all bounded linear operators on a Hilbert space and the canonical trace. Later various generalisations of this theorem, as well as related results, turned up. In particular, M.B. Ruskai extended the triangle inequality to the case of finite von Neumann algebras and bounded density matrices. So far, the strong subadditivity of entropy for semifinite algebras, even for bounded density matrices, has remained unproven, although it was stated as a conjecture by Ruskai. We present the strong subadditivity theorem for the Segal entropy, in a pretty general case of semifinite von Neumann algebra and arbitrary states. Some other properties of entropy mentioned above, are given here as consequences of the strong subadditivity theorem.

Since the strong subadditivity of entropy is fundamental in quantum information theory not only on account of its physical applications but also because of mathematical consequences, it seems important and interesting to have this property for the general case of Segal's entropy in semifinite von Neumann algebras.