Transition Probability of States on $*$-Algebras

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Abstract

Let f and g be states on a unital complex $*$-algebra A. The transition probability $P_A(f,g)$ is defined as the supremum of $|\langle\varphi_f,\varphi_g\rangle|^2$ over all $*$-representations π of A for which f and g are represented as vector states with corresponding vectors φ_f and φ_g. This number $d(f,g)=2-\sqrt{P_A(f,g)}$ is the Bures distance of f and g.

We present some results about the number $P_A(f,g)$ dealing with unbounded representations. The main technical ingredient is a description of $P_A(f,g)$ in terms of intertwiners related to the GNS representations of f and g. Under some regularity assumptions we prove : If A is the Weyl algebra and $f(a)=\mathrm{Tr}\ ta$ and $g(a)=\mathrm{Tr}\ sa$, then $P_A(f,g)=(\mathrm{Tr}\ |t^{1/2}s^{1/2}|)^2$. If A is an algebra of functions on a locally compact space X and f and g are given by densities η and ξ of a measure μ on X, then $P_A(f,g)=(\int_X\sqrt{\eta(x)\xi(x)}\,d\mu(x))^2$.