

# Noncommutative dimension and almost finiteness

Joachim Zacharias

University of Glasgow  
Joachim.Zacharias@glasgow.ac.uk

18 WORKSHOP, BĘDLEWO 2018

## Abstract

In recent years noncommutative dimension concepts such as nuclear dimension have proved to be very important in the classification of simple nuclear  $C^*$ -algebras. Finiteness of nuclear dimension has turned out to be the correct regularity condition to make the classification work. The other important regularity condition, now known to be equivalent to finite nuclear dimension is  $\mathcal{Z}$ -stability. One of the main problems in classification is now to prove that certain given algebras are actually classifiable e.g. have finite nuclear dimension or are  $\mathcal{Z}$ -stable. Some years ago Hiroki Matui introduced almost finiteness as a regularity condition for groupoids. Almost finiteness has turned out to be a very powerful tool to prove  $\mathcal{Z}$ -stability for crossed product algebras coming from discrete amenable groups acting minimally on compact metric spaces i.e. on abelian  $C^*$ -algebras. We extend the definition of almost finiteness to the setting of groups acting on noncommutative coefficient algebras. The resulting concept combines aspects of the definition of nuclear dimension and the abelian case. We obtain  $\mathcal{Z}$ -stability for almost finite actions. This is the joint work with Joan Bosa, Francesc Perera and Jianchao Wu.