18th Workshop: Noncommutative Probability, Operator Algebras, Random Matrices and Related Topics, with Applications

BABY TALKS

15-21.07.2018, Będlewo

1. Uwe Franz (Université de Bourgogne Franche-Comté)

   Lévy-Khinchin for quantum probabilists

   Abstract: We will sketch a quantum probabilistic proof of the Lévy-Khinchin formula that classifies convolution semigroups of probability measures on the real line and real-valued Lévy processes. In this lecture we will encounter generating functionals, Schürmann triples, and the symmetric Fock space with its fundamental noise operators.

2. Jacek Wesołowski (Politechnika Warszawska

   Quadratic harnesses as a link between Askey-Wilson polynomials and ASEPs

   Abstract: Quadratic harnesses are classical stochastic processess (X_t) determined by linear conditional expectations and quadratic conditional variances where conditioning is with respect to the past-future filtration of the process. Typically they are non-stationary Markov processes uniquely determined by five numerical constants, say θ ,η, τ, σ and γ. To a large extent their properties are determined by respective families of orthogonal martingale polynomials $(p_n(x,t))$ whose Jacobi matrices $\mathbf{J}_t$ are affine in the time variable t, i.e. $\mathbf{J}_t = t\mathbf{X} + \mathbf{Y}$ and infinite tridiagonal matrices $\mathbf{X}$ and $\mathbf{Y}$ satisfy the following γ-commutation equation: $\mathbf{XY} - q\mathbf{YX} = \mathbf{I} + \theta\mathbf{X} + \eta\mathbf{Y} + \tau\mathbf{X}^2 + \sigma\mathbf{Y}^2$. The family of quadratic harnesses is rather wide and includes e.g.: Wiener process, Poisson process, Levy-Meixner processes, quantum Bessel process, free Brownian motion, $q-$Gaussian process, Dirichlet process, bi-Poisson processes and others. Basic facts on quadratic harnesses will be presented in the lecture. In particular we will explain a connection between quadratic harnesses and Askey-Wilson processes which have been defined in 2010 as Markov processes with marginal distributions and transition probabilities being probability measures which orthogonalize Askey-Wilson system of polynomials. This connection in particluar introduces a new parametrization of Askey-Wilson polynomials - the new parameters are the five numerical constants θ ,η, τ, σ and γ identified above.

   ASEP is an acronym for asymmetric simple exclusion process and describes evolution of particles which are forced to move at random times one step to the left or right in sites $1, ..., N$ but only when the neighbouring site is not occupied; aditionally, particles are injected randomly from the left and right or can leave the system through border sites 1 or N. Then site occupations is a Markov process with the state space $\{0,1\}^N$. We will be interested in stationary (steady state) distribution of this process, i.e. the law of the site occupation $(\tau_1, \ldots, \tau_N)$ after a long run. It is well known that this distribution (actually, its generating function) can be determined through so called matrix ansatz formulated by Derrida, Evans, Hakim and Pasquier in 1993. This result is based on non-commutative calculations involving two infinite matrices $\mathbf{D}$ and $\mathbf{E}$ satisfying identity $\mathbf{DE} - q\mathbf{ED} = \mathbf{D} + \mathbf{E}$, where $q \in [0,1)$ is the intensity of jumps to the left, while 1 is the intensity of jumps to the right. This identity for $\mathbf{D}$ and $\mathbf{E}$ and the γ-commutation equation for $\mathbf{X}$ and $\mathbf{Y}$ (elements of the Jacobi matrix $\mathbf{J}_t$) form a theoretical base for the connection between ASEPs and quadratic harnesses (more precisely, Askey-Wilson processes) we are going to describe in the lecture. Actually, we will explain how the generating function of site occupations $(\tau_1, \ldots, \tau_N)$ can be represented in terms of moments of quadratic harness of the Askey-Wilson type. It will be also shown how this new represention can be used to study different properties of ASEPs. In particular, we will provide a relatively simple proof of LDP (large deviation principle) for average overall site occupancy as well as new formulas for so called density profiles. Finally we will show how the representation can be used to study asymptotic properties of ASEPs when N, the number of sites, tends to infinity. The talk is based on joint reserach with Wlodek Bryc (University of Cincinnati).