First integrals and exact solutions of some models from epidemiology via Hamiltonian approaches

Imran Naeem
Department of Mathematics, Lahore University of Management Sciences, Opposite Sector U, DHA, Lahore Cantonment 54792, Pakistan

We have investigated the first integrals and exact solutions of some models from epidemiology by utilizing the Hamiltonian approaches [1, 2, 3]. First, we study the optimal control problem of membrane filtration system. We employ the Noether approach for Hamiltonian formulation [1] to construct Noether symmetries and first integrals for the Hamiltonian system of first-order ordinary differential equations (ODEs) associated with the membrane filtration system. Then we utilize these first integrals to establish the exact solutions for the key variables of the model. Secondly, we analyze the optimal harvesting model in exact solutions perspective. We utilize the partial Noether approach for current value Hamiltonian [2] to construct partial Hamiltonian operators, first integrals and exact solutions for the partial Hamiltonian system of first-order ODEs associated with the harvesting model. Lastly, the two-stream model for tuberculosis and dengue fever governed by a dynamical system of first-order ODEs is investigated. We use the notion of artificial Hamiltonian [3] to express this dynamical system of first-order ODEs as a partial Hamiltonian system. We utilize the partial Noether approach [2] to construct partial Hamiltonian operators, first integrals and exact solutions for the partial Hamiltonian system of first-order ODEs associated with the two-stream model for tuberculosis and dengue fever. The exact solutions are plotted in order to observe the progression of disease.

REFERENCE

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