Elemental access to limit cycle existence in Biomath education

Quay van der Hoff
University of Pretoria, Pretoria, South Africa

Ansie Harding
University of Pretoria, Pretoria, South Africa

This paper originated from the desire to develop elementary calculus based tools to empower students, not necessarily with a strong mathematical background, to test predator-prey related models for boundedness of solutions and for the existence of limit cycles. There are several well-known methods available to prove, or disprove, the existence of bounded solutions to systems of differential equations. These methods rely on Liénard's theorem or using Dulac or Lyaponov functions. The level of mathematics required in the study of differential equations is not addressed in the courses presented on the first year level, and students in biology, ecology, economics and other fields are often not suitably equipped to perform these advanced techniques. The conditions under which a unique limit cycle exists in predator-prey systems is considered a primary problem in mathematical ecology [1, 2]. A great deal of mathematical effort has gone into trying to establish simple, yet general, theorems which will allow one to decide whether a given set of nonlinear equations has a limit cycle or not [3]. We introduce a method to first determine the boundedness of solution trajectories in such a way that the transformation to a Liénard system or the use of a Dulac function can be avoided. Once boundedness of trajectories has been established, the nature of the equilibrium points reduce to simple eigenvalue analysis. The Elemental Limit Cycle method (ELC) provides elementary criteria to evaluate the nature of the pivotal functions of a system which will indicate boundedness and may be applicable to more general models. Keyword: Elemental Limit Cycle Method, Biomath education, predator-prey, boundedness, limit cycles.

REFERENCE

[1] May RM, Limit cycles in predator-prey communities, Science, 177, 900-902 (1972)

[2] May RM, Stability and Complexity in Model Ecosystems, Princeton University Press, Princeton, NJ (1974)

[3] Enns RH, McGuire GC, Nonlinear Physics with Mathematica for Scientists and Engineers, Birkhauser, Boston (1997)