THE FRONTIER OF QUANTUM DYNAMICS

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Abstract. The mathematical scope of the program is within noncommutative geometry, quantum groups, graph C*- algebras, KK-theory, index pairing, C*-dynamical systems and crossed products, Dirac operators, spectral triples, curvature, and quantum Gromov-Hausdorff distance. There are a number unifying principles which establish connections among the proposed topics of the meeting. For example, dynamical systems link together graph C*-algebras and noncommutative metric geometry. We aim to construct quantum metric geometries of crossed products and graph algebras relating the Lipschitz norm and Dirac-operator approaches. This links the metric and spectral features of noncommutative geometry. A key research focus of the proposed activities is to make precise the impact of metric and spectral noncommutative geometry on noncommutative topology. Graph C*-algebras are an abundant source of examples which test the bridges between the topics of the proposed conference.

Titles and abstracts

Paul F. Baum (Penn State University, State College, USA), 12-19.12
Noncommutative geometry impacts representation theory
This talk will first give a precise definition of extended quotient. Next, the talk will indicate how extended quotient enters into the representation theory of Lie groups and into the representation theory of reductive p-adic groups. For Lie groups this is due to Mackey–Higson–Afgoustidis. For reductive p-adic groups this is due to Aubert–Baum–Plymen–Solleveld.

Andrzej Borowiec (Uniwersytet Wrocławski, Poland), 11-13.12
Unified description of κ-deformations of inhomogeneous orthogonal groups and their twisted extensions
We study Hopf-algebraic κ-deformations of all inhomogeneous orthogonal Lie algebras $iso(\mathfrak{g})$ as written in a tensorial and unified form. Such deformations are determined by a vector τ which for Lorentzian signature can be taken time-, light- or space-like. We describe real forms with connection to the metric's signatures and their compatibility with the reality conditions for the corresponding κ-Minkowski (Hopf) module algebras. Secondly, h-adic vs q-analog (polynomial) versions of deformed algebras including specialization of the formal deformation parameter κ to some numerical value are considered. In the latter case the general covariance is lost and one deals with an orthogonal decomposition. The last topic treated in this talk concerns twisted extensions of κ-deformations as well as the description of resulting noncommutative spacetime algebras in terms of solvable Lie algebras. We found that if the type of algebra does not depend on deformation parameters then specialization is possible.

Ludwik Dąbrowski (SISSA, Trieste, Italy), 8-13.12
Almost commutative geometry of the Standard Model
A non-commutative C*-algebra is commonly regarded as the algebra of continuous functions on a 'quantum space'. Its smooth and metric structures can be described in terms of a spectral triple that involves an analogue of the Dirac operator. I will explain how the Standard Model of fundamental particles can be interpreted as the almost commutative geometry. The exterior part is the canonical spectral triple on a spin manifold, and the finite inner part is a quantum analogue of the de Rham–Hodge spectral triple.

Michał Eckstein (Uniwersytet Gdański, Poland), 11-12.12
An invitation to the spectral action
The spectral action is a primary tool to study the dynamics in the context of spectral triples. Motivated by the celebrated Least Action Principle, it plays a pivotal role in physical applications of noncommutative geometry a la Connes. Its formulation is very simple, but this simplicity is deceiving - explicit computations are rather cumbersome and no general methods are available. In my talk, I will highlight the assets and trouble spots of the spectral action computations. General results will be illustrated with a few examples and possible physical applications. The talk is based on a recent book with Bruno Iochum.

Carla Farsi (University of Colorado, Boulder, USA), 12-16.12
Matui's HK conjecture for higher-rank Renault–Deaconu groupoids and ample groupoids
Ample groupoids and their C*-algebras have recently attracted a lot of attention. Higher-rank graph and Renault–Deaconu groupoids are particularly interesting examples of ample groupoides. In this talk I will outline some recent results on ample groupoids that are related to Matui's HK (Homology-K-Theory) conjecture. This is joint work with Kumjian, Pask, and Sims.

Piotr M. Hajac (IMPAN, Warsaw, Poland), local
From pushouts of graphs to pullbacks of graph algebras
We search for a new concept of graph morphism that would ensure that the assignment of graph algebras to graphs becomes a contravariant functor translating pushouts of graphs into pullbacks of graph algebras. The case of an injective morphism between row-finite graphs is solved by a known concept of admissible subgraph. The non-injective case is motivated by natural and highly non-trivial examples from noncommutative topology (e.g., quantum weighted projective spaces). To accommodate this naturally occurring non-injectivity, we replace the standard idea of mapping vertices to vertices and edges to edges by the more flexible idea of mapping finite paths to finite paths. (Based on joint works with Alexandru Chirvasitu, Sarah Reznikoff and Mariusz Tobolski.)

Byung-Jay Kahng (Canisius College / SUNY, Buffalo, USA), 1.11-31.12
Manageability of multiplicative partial isometries and quantum groupoids
It is known that in the theory of quantum groupoids of separable type (developed by the author and Van Daele), there arise certain multiplicative partial isometries that behave in a similar manner as the multiplicative unitaries (in the sense of Baaj–Skandalis) associated with a quantum group. In this talk, I will give and explain some algebraic conditions that axiomatically determine a multiplicative partial isometry W. Then we will also consider the manageability condition for W. Starting from the multiplicativity and the manageability, we can construct most of the quantum groupoid structure, including a C*-algebra A with a comultiplication Δ, base C*-algebras B_s and B_t, as well as the antipode map and its polar decomposition. Its dual quantum groupoid can be also constructed. We will then turn to exploring an ongoing project (with Woronowicz) concerning the partial isometries that are "adapted" to our given manageable multiplicative unitary W.

Paweł Kasprzak (Uniwersytet Warszawski, Poland), local
Noncommutative Furstenberg boundary
In my talk the concept of a G-boundary action for a discrete quantum group G on a C*-algebra A will be introduced. It turns out that there exists the largest (with respect to G-covariant embeddings) G-boundary C*-algebra that we call a noncommutative Furstenberg boundary of G. The unique trace property of $C^*(G)$ for a discrete quantum group G of Kac type will be shown to be implied by the the faithfulness of the action on noncommutative Furstenberg boundary of G. I will show (or rather just say) that the Gromov boundary of a generic free orthogonal quantum group is a boundary action in our sense. Joint work in progress with Kalantar, Skalski and Vergnioux.

Jacek Krajczok (IMPAN, Warsaw, Poland), local
Coamenability of type I locally compact quantum groups via convolution operators
We say that a locally compact quantum group is type I if its universal C* algebra (which is the universal version of the C*-algebra of continuous functions vanishing at infinity on the dual group) is type I. This class of quantum groups can be though of as an intermediate step between compact and general locally compact quantum groups; they are significantly more general than compact ones, but still have tractable representation theory. If G is a compact quantum group, then one can introduce certain convolution operators on the Hilbert space $\ell^2(Irr(G))$, properties of which allow us to detect whether G is coamenable. During the talk I will outline some results concerning generalization of such criterion to the case of type I locally compact quantum groups.

Dan Kučerovský (University of New Brunswick, Fredericton, Canada), 30.09-31.12
Relative double commutans in coronas of separable C-algebras*
Given a subalgebra A of a C*-algebraic corona algebra, one can define a relative commutant. In several cases, it is known that the double relative commutant of A is A itself. We show that this holds true in a class of corona algebras for separable subalgebras A that are unital in a suitable sense. The class of corona algebras considered is the class of coronas of stable, separable, simple, nuclear C*-algebras. Separability is an essential condition, as shown by some counterexamples. (Joint work with Martin Mathieu, QUB.)

Frédéric Latrémolière (University of Denver, USA), 8-10.12
Gromov-Hausdorff convergence for some C-dynamical systems*
While developing a noncommutative analogue of the Gromov–Hausdorff distance for C*-algebras, we observed that, under some assumptions, our new metric interacts rather nicely with (semi)group actions. A natural way to capture this phenomenon is to introduce a covariant version of our Gromov–Hausdorff distance and prove a form of the Arzela–Ascoli theorem relating the covariant and the standard Gromov–Hausdorff convergence. Certain interesting issues arise regarding the completeness of the covariant Gromov–Hausdorff metric. We will discuss this topic, present some applications, and point out how this work is a component of our more recent work on the convergence for spectral triples.

Kang Li (IMPAN, Warsaw, Poland), local
Diagonal dimension for C-pairs*
We will introduce the notion of diagonal dimension for diagonal pairs of C*-algebras in the sense of Kumjian, and will compare it with the usual nuclear dimension for C*-algebras. For instance, the Jiang-Su algebra $\mathcal{Z}$ admits a diagonal MASA D such that the diagonal dimension of $(\mathcal{Z}, D)$ is equal to n for any given natural number n even though the nuclear dimension of $\mathcal{Z}$ is equal to 1. We also show that the diagonal dimension of a uniform Roe algebra with respect to the standard diagonal is equal to the asymptotic dimension of its underlying metric space. Finally, we will discuss its relation to the dynamic asymptotic dimension of groupoids introduced by Guentner, Willett and Yu and the (fine) tower dimension of topological dynamical systems introduced by Kerr.

Tomasz Maszczyk (Uniwersytet Warszawski, Poland), local
Milnor idempotents through Toeplitz projections
The K-theory of classical complex projective spaces was computed first time by Atiyah and Todd with use of tools which are not available in noncommutative geometry. In particular, their computation determines also the generators in terms of associated vector bundles. Recently, the K-theory of a quantization of complex projective spaces, defined as a replacement of all discs in the classical pushout by quantum discs, has been computed by Hajac-Nest-Pask-Sims-Zieliński. The generators have been found by Sheu in terms of Toeplitz projections. The main difficulty in relating his result with the classical result of Atiyah and Todd is that the Toeplitz projections do not admit a classical limit. We explain the failure of the classical limit of Toeplitz projections by showing that they are subject to a noncommutative real blow-up

of the center of a disc in the classical limit of the quantum disc deformation. We overcome this difficulty in the case of a complex projective plane, reobtaining the result of Atiyah and Todd in a way admitting a quantization together with an explicit homotopy between a quantized Milnor type idempotent, coming from a clutching construction, and the Toeplitz type projection of Sheu. (Joint with C. Farsi, P. M. Hajac, B. Zieliński.)

Ryszard Nest (University of Copenhagen, Denmark), 8-12.12
Around the functional equation
The functional equation for the Riemann zeta function is based on analysis of asymptotic behaviour for $t \sim 0$ of expression like $\mathrm{Tr}(\exp(-tD^2))$, where D is, say, an elliptic operator on a smooth closed manifold M. In particular, it depends heavily on the the fact that the expressions like $\mathrm{Tr}(\exp(-tD^2))$ have Melin transform which is holomorphic on a subspace of the complex plane of the form $\mathrm{Re}(z) > C$, which is a consequence of finite dimensionality of M. We will construct an analogue of the meromorphic extension of the Riemann zeta function and prove the corresponding functional equation in the infinite dimensional limit case. We will sketch some work in progress which give applications of these constructions to local index formulas for operators associated to infinite dimensional physical systems.

Tatiana Shulman (IMPAN, Warsaw, Poland), local
Continuity of spectral radius on C-algebras*
We will address some problems about C*-algebras involving the spectral radius function. We will discuss connections with Balackadar's projectivity theory for C*-algebras.

Andrzej Sitarz (Uniwersytet Jagielloński, Cracow / IMPAN, Warsaw, Poland), 9.12, 12-13.12
Multitwisting real spectral triples
The notion of spectral triple with reality structure is generalized to multitwisted real spectral triples, the class of which is closed under the tensor product composition. In particular, we introduce a multitwisted order one condition (characterizing the Dirac operators as an analogue of first-order differential operator). This provides a unified description of almost all known examples. Based on joint work with Ludwik Dąbrowski.

Piotr M. Sołtan (Uniwersytet Warszawski, Poland), local
Podleś spheres for the braided quantum $SU(2)$
The construction of "quantum spheres" was first accomplished by Podleś , starting with the quantum group $SU_q(2)$ for real q such that $0 < |q| < 1$. Recently, a version of this quantum group was defined by Kasprzak, Meyer, Roy and Woronowicz for complex q satisfying $0 < |q| < 1$. Here the crucial difference is that, if q is not real, the comultiplication takes values in the "braided tensor product" of the C*-algebra $C(SU_q(2))$ with itself. I will discuss the result that, despite this difference, the quantum spheres in the braided case (q not real) are exactly the same as those found by Podleś for $|q|$.

Mariusz Tobolski (IMPAN, Warsaw, Poland), local
Local triviality of noncommutative principal bundles
The local-triviality dimension is an invariant of an action of a compact (quantum) group on a unital C*-algebra. It plays the role of local triviality of a principal bundle in the noncommutative setting. In this talk, we define the aforementioned dimension and show that, in the classical case, it is precisely the minimal cardinality of the trivializing cover of a principal bundle. Next, we construct a universal example of an action with a given local-triviality dimension for any compact (quantum) group G and discuss its equivariant and non-equivariant K-theory. This construction leads to a candidate for a noncommutative classifying space for a compact (quantum) group.

Elmar Wagner (Universidad Michoacana de San Nicolás de Hidalgo, Morelia, México), 9-14.12
The Dolbeault-Dirac operator on the quantum quadratic $SO(5)/SO(2) \times SO(3)$
The aim of the talk is to present an explicit construction of Dirac operators on irreducible quantum flag manifolds using the quantum tangent space approach of I. Heckenberger and S. Kolb. In the case of the quantum quadratic $SO(5)/SO(2) \times SO(3)$, it will be shown how to compute the spectrum of the Dolbeault-Dirac operator.

Bartosz Zieliński (Uniwersytet Łódzki, Poland), 12.12
The K-theory of multipullback quantum projective spaces
We recall in detail (with classical motivation) definition of multipullback quantum projective spaces both as multipullbacks as well as U(1)-equivariant subalgebras of quantum multipullback odd spheres. We prove that that the K-groups of multipullback quantum complex projective spaces and odd spheres coincide with their classical counterparts. Let us remark that these K-groups remain the same for more general twisted versions of our quantum odd spheres and complex projective spaces.

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