ON SOME PROBLEMS OF K. BORSUK CONCERNING HOMOTOPY DOMINATIONS

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Recall that a map $f : X \rightarrow Y$ is a *homotopy domination* if there exists a map $g : Y \rightarrow X$ such that $fg \simeq id_Y$. Then we write $X \geq Y$, and we say that Y is homotopy dominated by X. In the sequel, as usually, every polyhedron is finite and every ANR is compact.

In this talk I will discuss the longstanding Borsuk's problem: *Is it true that two ANR's homotopy dominating each other have the same homotopy type?* [K. Borsuk, "Theory of Retracts", 1967], and closely related open question: *Is it true that each homotopy domination of a polyhedron over itself is a homotopy equivalence?* [J. Dydak, A. Kadlof, S. Nowak, 1981].

Given a polyhedron P, one may ask: *Is it true that each sequence $P \geq X_1 \geq X_2 \geq \ldots$ contains only finitely many homotopy dominations which are not homotopy equivalences?*, or: *Does there exist an integer l_P (depending only on P) such that each sequence of this kind contains only $\leq l_P$ homotopy dominations which are not homotopy equivalences?* In the second case, P have *finite depth* (this notion was introduced by K. Borsuk in 1979).

(By the result of J. West [Ann. of Math. 1975], we may use the notions "polyhedron" and "ANR" interchangeably.)

I will present my latest results concerning these problems and some related interesting questions on finitely presented groups.