Smooth self-maps of semi-simple Lie groups realizing the least number of n-periodic points

Jerzy Jezierski
Warsaw University of Life Sciences (SGGW), Warszawa POLAND

August 19, 2019

We consider a map $f : M \to M$, where M is a compact connected smooth manifold, $\dim M \geq 3$. We fix a number $n \in \mathbb{N}$. The minimal number of n-periodic points in the homotopy class of f, i.e. $\min_{g \sim f} Fix(g^n)$, depends on which homotopy class we consider: continuous or smooth. However in the case of self-maps of tori, the unique compact Lie groups, the two numbers are always the same. The last exactly means that the least number of n-periodic points can be realized by a continuous map. On the other hand for each noncommutative Lie group the power map $f(z) = z^k$, for any $k \geq 2$ and an n with sufficiently many divisors, is a counter-example.

Here we consider the problem *for which self maps of Lie groups the equality holds for all iterations.* The necessary condition is given by eigen-values of a cohomology homomorphism introduced by Haibao Duan [1]. We extend the class of semi-simple Lie groups when this condition is also sufficient for a smooth realizability of the least number of $n-$periodic points.

References

[1] H. Duan *A characteristic polynomial for self-maps of H -spaces.* Quart. J. Math. Oxford Ser. (2) 44 (1993), no. 175, 315-325.

[2] J. Jezierski, *Least number of periodic points of self-maps of Lie groups* Acta Math. Sinica, English Series 2014, Vol. 30, nr 9, s. 1477-1494

[3] J. Jezierski, *When a smooth self-map of a semi-simple Lie group can realize the least number of periodic points* Sci. China Math. 60 (2017), no. 9, 1579-1590.