Algebra, Geometry and Topology of the Riordan group

Manuel Alonso Morón
Universidad Complutense de Madrid, España
(All of this is a joint work with **Ana Luzón** and some parts with other authors.)

In this talk I will describe our contribution (in the framework described by the title of the talk) to the study of a combinatorial device known under the name of *the Riordan goup*. It is a group of infinite, invertible, lower triangular matrices with the usual matrix product. The better known celebrity among its elements is Pascal's triangle.

I will start describing our particular way of getting all its elements, called Riordan matrices, by means of Banach's Fixed Point Theorem. After that I will give the induced iterative process to construct all of them.

Later, I focus on the computation of the corresponding series of commutators (the derived series), the construction of all its involutions and on the algebraic recognition of the subgroup generated by them.

In the main part of the talk I will point out, first, how this group can be considered both as a pro-Lie group and as a Frechet infinite dimensional Lie group (in the sense of Milnor). I will also describe the corresponding Lie Algebra. I will also comment how the method of characteristics to solve quasi-linear partial differential equations of first order, allows us to describe/compute the exponential map in this Lie Group. After pointing out that the subset of non-trivial involutions in this group has a natural structure of sub-manifold, I will describe families of linear first order differential equations with *time-reversal symmetries* in any euclidean space. Also families of bivariate partial differential equations with the same time-reversal property. I will try to develop some examples, mainly those where Pascal's triangle is involved.

If time permits I will also say some words about topological aspects of this group mainly those related to the fact that *The substitution group of formal power series*, which is better known in the topological context, can be easily considered as a subgroup of The Riordan group.