HYPERSPACES OF INFINITE COMPACTA WITH FINITELY MANY ACCUMULATION POINTS

PAWEŁ KRUPSKI AND KRZYSZTOF OMILJANOWSKI

Let $\mathcal{A}_n(X)$ ($\mathcal{A}_\omega(X)$) be the Vietoris hyperspace of infinite compact subsets of a metric space X which has at most n (finitely many, resp.) accumulation points. Hyperspace $\mathcal{A}_n(X)$ is an $F_{\sigma\delta}$ subset of the hyperspace $\mathcal{K}(X)$ of all compacta in X.

If X is a nondegenerate locally connected metric continuum then $\mathcal{A}_n(X)$ is an absolute retract for all $n \in \mathbb{N} \cup \{\omega\}$. If $X = J = [-1, 1]$ or $X = S^1$, hyperspaces $\mathcal{A}_n(X)$ are characterized as $F_{\sigma\delta}$-absorbers in Hilbert cubes $\mathcal{K}(J)$ and $\mathcal{K}(S^1)$, respectively. Consequently, they are homeomorphic to the linear space $\{(x_k) \in \mathbb{R}^\mathbb{N} : \lim x_k = 0\}$ with the product topology. If X is a locally connected nondegenerate continuum and $p \in X$ is a point of order ≥ 2, then $\mathcal{A}_1(X, p) := \{K \in \mathcal{A}_1(X) : A' = \{p\}\}$ is also an $F_{\sigma\delta}$-absorber in Hilbert cube $\mathcal{K}(X)_p := \{K \in \mathcal{K}(X) : p \in K\}$.

The hyperspaces $\mathcal{A}_\omega(X)$ for X being a Euclidean cube, the Hilbert cube, the m-dimensional unit sphere S^m, $m \geq 1$, or a compact m-manifold with boundary in S^m, $m \geq 3$, are true $F_{\sigma\delta\sigma}$-sets which are strongly $F_{\sigma\delta}$-universal in the respective hyperspaces of all compacta.

Wrocław University of Science and Technology

University of Wrocław