RECENT PROGRESS ON A CONJECTURE OF BING AND BORSUK

Abstract. (Joint with J Bryant) In his 1994 ICM talk Shmuel Weinberger, inspired by work of Edwards, Quinn, Cannon, Bryant-F.-Mio and himself, conjectured the existence of a new collection of spaces with many of the properties of topological manifolds. The authors have constructed spaces in dimensions $n \geq 6$ satisfying many parts of Weinberger's conjecture. Our spaces are finite dimensional and locally contractible. They have the local and global separation properties of topological manifolds, satisfying Alexander duality both locally and globally. More technically, they are integral ENR homology manifolds. They are homogeneous, meaning that for every x and y in a component of one of these spaces there is a homeomorphism carrying x to y. There are countably many topologically distinct examples in the homotopy type of any closed, simply connected manifold. In particular, these spaces are counterexamples to a well-known conjecture of Bing and Borsuk. The situation for nonsimply connected manifolds is more complicated. In particular, none of our new spaces can have the homotopy type of a torus. It is an interesting open question whether any of these new spaces can be aspherical. In high dimensions, the h- and s-cobordism theorems hold for these topologically exotic manifolds.