
On the complement of a countable union of Z -set continua in the Hilbert cube

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We are interested in spaces that are described in the title. We additionally assume that the removed continua are pairwise disjoint. It turns out that, if two such complements are homeomorphic then up to a permutation the shapes of the removed continua are the same. This fact generalizes (for continua) the easy part of Chapman Complement Theorem. A counterpart of the more difficult part of the Champan theorem is, in general, false. However, a certain “discrete” version of such a counterpart seems plausible.

Our approach goes back to works of van Mill (and others). van Mill used such a complement space to find a countable dense homogeneous space which contains a rigid open dense subset, that is, the identity is the only homeomorphism of the open set. Moreover, van Mill’s example is a “square root” of the Hilbert space (that is, its square is homeomorphic to the separable infinite-dimensional Hilbert space). We are able to adjust van Mill’s construction and exhibit such an example in the 3-dimensional cube.

However, the question of whether such an object exists within local compacta is unsolved.

References

- [1] J. van Mill, *A countable dense homogeneous space with a dense rigid open subspace*, Fund. Math. **201** (2008), 91–98.