Testing equality of mean vectors with block-circular covariance matrices

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Abstract

While the likelihood ratio test for the test of equality of mean vectors, when the covariance matrices are only assumed to be positive-definite, is a common test in Multivariate Analysis and although there is also some work done on tests of equality of means with structured covariance matrices, namely with block-circular matrices [1], there is not much work done on the test of equality of mean vectors when some structure is assumed for the covariance matrices. In this presentation the author develops the likelihood ratio tests for the equality of mean vectors when the covariance matrices have a common circular or circulant structure. Besides obtaining the likelihood ratio statistic, also its exact distribution is characterized in terms of identifying it with the distribution of a product of independent Beta random variables. It is then shown that in some cases this distribution may have a very manageable form, making it very easy and quick to compute exact quantiles and p-values for these cases. For the other cases where this exact distribution is not possible to be obtained in a manageable form, very sharp near-exact distributions are developed. These will enable the quick computation of near-exact quantiles and p-values and are shown to yield very sharp approximations to the exact distribution, exhibiting a clear asymptotic behavior not only for increasing sample sizes but also, and opposite to common asymptotic distributions, for increasing numbers of subsets of variables and increasing numbers of variables in each subset, which amounts to be asymptotic also for the overall number of variables involved. These near-exact distributions are built on a factorization of the characteristic function of the negative logarithm of the likelihood ratio statistic and besides having the above desirable properties they also exhibit a very good performance, that is, a very good proximity to the exact distribution, for very small samples, with this proximity even improving as the overall number of variables involved increases.

Keywords

Beta random variables, Characteristic function, Exact distribution, Likelihood ratio test, Near-exact distributions.

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References

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