

# The comparison of the estimators of banded Toeplitz covariance structure

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## Abstract

We consider the model of experiment in which several characteristics are measured or the measurements are repeated in time. We assume that the covariance matrix of the characteristics/time points is unknown, but has the banded Toeplitz structure. The problem of estimating such a covariance structure was considered by e.g. Cui et al. (2016) [1], Filipiak et al. (2018) [2].

We discuss two methods that aim to deal with singularity and numerical ill-conditioning of the estimators of covariance matrix, especially under high-dimensional regime.

The method of estimation of banded Toeplitz covariance matrix based on a shrinking, described by Ledoit and Wolf (2004) [3] for unstructured covariance matrices is proposed. The estimators obtained with the use of the proposed method are then compared with the estimators obtained by the projection of sample covariance matrix onto the asymptotic cone of the nonnegative definite Toeplitz matrices (Filipiak et al., 2018). For this purpose simulation studies concerning bias and risk are conducted for several sets of parameters.

## Keywords

Covariance matrix, Estimation, Projection, Shrinkage, Toeplitz matrix.

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