International Conference on Multivariate and Mixed Linear Models and Big Data Analytics & Applications Symposium

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Book of Abstracts

April 29 – May 4, 2019
Będlewo, Poland

Edited by

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Contents

Part I

Introduction

The International Conference on Multivariate and Mixed Linear Models, MMLM 2019, will be held on Arpil 29 - May 4, 2018, at Będlewo near Poznań. Będlewo is the Mathematical Research and Conference Center of the Polish Academy of Sciences. It will sum up the research meetings on *Multivariate Linear Models and Designs of Experiments*, *Sufficient and Optimal Statistical Procedures in Mixed Linear Model*, *Planning and Analysis of Tensor-Experiments*, *Neighbour Designs and Crossover Designs*, *Mixed and Multivariate Models*, *Multivariate Linear Models*, and *Multivariate and Mixed Linear Models*, which have been held in Będlewo since 2009, usually twice per year. As the result of discussions of the research meetings over 50 peer reviewed articles and one book has been published or submitted to journals.

The aim of the conference is to bring together researchers sharing an interest in statistical procedures in multivariate and mixed linear models together with their applications in economy, agriculture or engineering. One of the objectives is to apply tensor operators to statistical modeling of highly complex data which arises in almost all fields of contemporary science and technology. The main topics of the meeting are optimal estimation and hypothesis testing on covariance structure. The participants will present the results and current developments of their research. Mutual discussions on possible solutions to the presented problems are main and significant part of the meeting.

It is expected that selected papers presented on the conference will be published, after refereeing, in Lecture Notes in Statistics (Springer).

The conference will include invited talks given by

- S. Ejaz Ahmed (Canada),
- Carlos Coelho (Portugal),
- Tõnu Kollo (Estonia),
- Jianxin Pan (UK),
- Akimichi Takemura (Japan).

We are pleased to announce that the special Symposium on Big Data Analytics & Applications will be held just after the conference, on Saturday, 4th of May. The main speaker of this seminar is Prof. S. Ejaz Ahmed from Brock University, Canada. In the second part of the symposium practical aspects of Big Data will be discussed.

MMLM 2019 Organizers

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Part II

Program

Program

Sunday, April 28, 2019

14:00 – 19:00 Registration
19:00 – Dinner

Monday, April 29, 2019

8:00 – 9:00 Breakfast

9:55 – 10:00 *Opening*
10:00 – 11:15 T. Kollo: *Parameter estimation for skew elliptical distributions*

11:15 – Coffee Break

11:45 – 12:15 K. Filipiak: *Some comments on maximum likelihood estimation under various statistical models*

13:00 – Lunch

16:00 – 17:00 I. Žežula: *Testing means in two-level compound symmetry multivariate data*

17:00 – Coffee Break

17:30 – 18:00 M. Mokrzycka: *Covariance matrix approximation in doubly multivariate models*
18:00 – 18:30 M. Janiszewska: *Maximum likelihood estimation in applications*

19:00 – Dinner

Tuesday, April 30, 2019

8:00 – 9:00 Breakfast

10:00 – 11:15 C. Coelho: *Testing equality of mean vectors with block-circular covariance matrices*

11:15 – Coffee Break

11:45 – 12:45 A. Kozioł, R. Zmyślony: *Simultaneous testing hypotheses in models with blocked compound-symmetric covariance structure*

13:00 – Lunch

16:00 – 16:30 A. Szczepańska-Álvarez: *MLE of separable covariance matrix structure – simulation study*
16:30 – 17:00 D. Klein: *Growth curve model with compound symmetry structure*

17:00 – Coffee Break

17:30 – 18:00 A. Mieldzioc: *The comparison of the estimators of banded Toeplitz covariance structure*
18:00 – 18:30 M. John: *Testing hypotheses about banded Toeplitz covariance structure under the high-dimensional models*

19:00 – Barbecue

Wednesday, May 1, 2019

8:00 – 9:00 Breakfast

10:00 – 11:15 A. Takemura: *Holonomic gradient method for multivariate distribution theory*

11:15 – Coffee Break

11:45 – 12:45 S. Puntanen: *A review of the linear prediction sufficiency in the linear model with new observations*

13:00 – Lunch

16:00 – 16:30 R. Kala: *On a simplified approach to estimation in experiments with orthogonal block structure*
16:30 – 17:00 M. Prus: *Optimal designs in multiple group random coefficient regression models*

17:00 – Coffee Break

17:30 – 18:00 A. Markiewicz: *On sufficiency of quadratically sufficient statistics in possibly mixed model*
18:00 – 18:30 J. Hauke: *On properties of Lee's bivariate spatial association measure*

19:00 – Dinner

Thursday, May 2, 2019

8:00 – 9:00 Breakfast

9:00 – 15:00 Excursion

15:00 – Lunch

16:00 – 17:15 J. Pan: *Case-deletion diagnostics for linear mixed models*

17:15 – Coffee Break

18:00 – 18:45 V. Witkovský: *Computing the exact distribution of selected test statistics in multivariate analysis*

19:30 – Conference Dinner

Friday, May 3, 2019

8:00 – 9:00 Breakfast

10:00 – 10:45 D. von Rosen: *The growth curve model under high dimensions*

10:45 – 11:15 J. Pielaszkiewicz: *Comments on the recursive formula on spectral moments of Wishart matrix and on the Rao score test statistics under model with BCS structured covariance matrix*

11:15 – Coffee Break

11:45 – 12:15 D. Uwamariya: *Large deviation probabilities of condition numbers of sample covariance matrices*

12:15 – 12:45 E. Umunoza Gasana: *Distribution of the squared first antieigenvalue*

13:00 – Lunch

16:00 – 17:15 S. E. Ahmed: *Penalty, pretest and shrinkage strategies in GLM*

17:15 – Coffee Break

17:30 – 18:00 P. Pokarowski: *Optimal penalty for GIC in subgaussian linear models*

18:00 – 18:30 H. Ćwiek-Kupczyńska: *Semantic description of linear mixed model analysis*

19:00 – Dinner

Saturday, May 4, 2019

BIG DATA ANALYTICS & APPLICATIONS SYMPOSIUM

10:00 – 10:15 R. Zmyślony: *Welcome/Introduction*
10:15 – 11:00 S. E. Ahmed: *Big Data Analytics: challenges and opportunities*

11:00 – Healthy break

11:30 – 12:30 S. J. H. Ahmed: *Identifying Advertisement Opportunities based on Integrating Public Data*

12:30 – Lunch break

14:00 – 15:00 S. E. Ahmed: *High Dimensional Data Analysis: Model Selection and Post Prediction - I*

15:00 – Healthy break

15:30 – 16:15 S. E. Ahmed: *High Dimensional Data Analysis: Model Selection and Post Prediction - II*

16:15 – Panel Discussion - Where to from here?

18:00 – Dinner

Part III

Big Data Analytics & Applications Symposium

Big Data Analytics & Applications

S. Ejaz Ahmed

Brock University, St. Catharines, Ontario, Canada

Abstract

There are hosts of buzzwords in today's data-centric world, and especially in digital and print media. We encounter data in every walks of life, and for analytically and objectively-minded people, data is everything. However, making sense of the data and extracting meaningful information from it may not be an easy task. We come across buzzwords such as big data, high dimensional data, data visualization, data science, and open data without a proper definition of such words. The rapid growth in the size and scope of data sets in a host of disciplines has created a need for innovative statistical strategies analyzing such data. For example, many private and public agencies are using sophisticated data mining strategies and/or big data analytics to reveal patterns based on collected information. Some examples of big data that have prompted demand are digital marketing, customer service standards, gene expression arrays, social network modeling, clinical, genetics and phenotypic data.

The purpose of this workshop is bi-fold.

1. We focus on estimation of model parameters and prediction based on high dimensional data (HDD). In classical regression context, we define HDD where number of predictors (p) are larger than the sample size (n). A host of the classical techniques are available when $p < n$ to reveal the data story. However, the existing classical strategies are not suited for providing solutions in the case of HDD. Over the past two decades, many penalized regularization approaches developed to perform variable selection and estimation simultaneously. We discuss model selection and post prediction strategies providing a better trade-off between model prediction and model complexity.
2. Most organizations have the ability to gauge their own individual performance through benchmarking and set KPI/Metrics. In the realm of ecommerce and digital marketing, the opportunity to incorporate consumer behavior will provide valuable insight on how the market demand could influence an organization's profitability. In this case, we will use an example of a retailer who has both an online and physical store. We will use an Application Programming Interface (API) where we will make a request and get information from Spotify streaming service to see what albums are in demand on their platform. This information will give us a method to assess inventory and consider what albums should be advertised both online and in the physical store. We will use Postman application to show the basics of an API call and then use Python to make a more detailed call. After we have ingested the data from Spotify, a medoid clustering approach is used to characterize album data. The medoid clustering is robust strategy than K-means approach; it is less sensitive to outliers. Based on the 3D clustering approach, we can identify which albums are in demand and assess our risk through Spotify's popularity

metric. Finally, we extract the albums based on the cluster and start targeting to reflect the market trends, potentially shift our inventory to mitigate risk and create a strategy on niche albums.

The suggested methodology is quite general in nature and has potential for implementation to a host of marketing scenarios.

S. Ejaz Ahmed

Dr. S. Ejaz Ahmed is Professor of Statistics and Dean of the Faculty of Math and Science at Brock University, Canada. Previously, he was Professor and Head of the Mathematics and Statistics Department at the University of Windsor, Canada and University of Regina, Canada as well as Assistant Professor at the University of the Western Ontario, Canada. He holds adjunct professorship positions at many Canadian and International universities. He has supervised numerous Ph.D. and Master Students, and organized several international workshops and conferences around the globe. He is a Fellow of the American Statistical Association. His areas of expertise include big data analysis, statistical learning, and shrinkage estimation strategy. Having authored several books, he edited and co-edited several volumes and special issues of scientific journals. He is Technometrics Review Editor for past ten years. Further, he is Editor and associate editor of many statistical journals. Overall, he published more than 175 articles in scientific journals and reviewed more than 100 books. Having been among the Board of Directors of the Statistical Society of Canada, he was also Chairman of its Education Committee. Moreover, he was Vice President of Communications for The International Society for Business and Industrial Statistics (ISBIS) as well as a member of the "Discovery Grants Evaluation Group" and the "Grant Selection Committee" of the Natural Sciences and Engineering Research Council of Canada.

For more information see:
HTTPS://BROCKU.CA/MATHEMATICS-SCIENCE/MATHEMATICS/DIRECTORY/SYED-EJAZ-AHMED/

S. Jazib H. Ahmed

S. Jazib H. Ahmed is currently a Data Analyst at Fanxchange, an online Canadian secondary ticket exchange. He previously worked in the automotive and education industry. He has successfully implemented analytics in multiple organizations and has dealt with the stigma where decision-making is solely based on industry experience rather than incorporating a scientific data driven approach. He is an avid researcher on the industryŠs best practices where he incorporates those methods in its application in marketing.

Part IV

Invited Speakers – Abstracts

Penalty, pretest and shrinkage strategies in GLM

S. Ejaz Ahmed

Brock University, St. Catharines, Ontario, Canada

Abstract

I will consider the estimation and prediction strategies in the context of generalized linear models when there are a number of predictors are available to Data Scientist, and some of them may have no and/or weak affect in predicting the response variable. We scrupulously investigate the relative performances of shrinkage, pretest, and penalty estimators with respect to the full model estimator. We examine and appraise the asymptotic properties of the non-penalty estimators. A Monte Carlo simulation study shows that the mean squared error (MSE) of an adaptive shrinkage estimator is comparable to the risk of the penalty estimators in many situations. Interestingly, when the dimension of the restricted parameter space is large, the shrinkage estimators perform better than the penalized likelihood estimators do. We illustrate the usefulness of the proposed strategies by applying to a real data set. Finally, we discuss how to extend these strategies to ultra high dimensional data.

References

[1] Ahmed, S.E. (2014). *Penalty, Shrinkage and Pretest Strategies: Variable Selection and Estimation*. Springer.
HTTPS://WWW.SPRINGER.COM/GP/BOOK/9783319031484.

Testing equality of mean vectors with block-circular covariance matrices

Carlos A. Coelho

[1] Universidade Nova de Lisboa, Portugal
[2] University of Pretoria, South-Africa

Abstract

While the likelihood ratio test for the test of equality of mean vectors, when the covariance matrices are only assumed to be positive-definite, is a common test in Multivariate Analysis and although there is also some work done on tests of equality of means with structured covariance matrices, namely with block-circular matrices [1], there is not much work done on the test of equality of mean vectors when some structure is assumed for the covariance matrices. In this presentation the author develops the likelihood ratio tests for the equality of mean vectors when the covariance matrices have a common circular or circulant structure. Besides obtaining the likelihood ratio statistic, also its exact distribution is characterized in terms of identifying it with the distribution of a product of independent Beta random variables. It is then shown that in some cases this distribution may have a very manageable form, making it very easy and quick to compute exact quantiles and p-values for these cases. For the other cases where this exact distribution is not possible to be obtained in a manageable form, very sharp near-exact distributions are developed. These will enable the quick computation of near-exact quantiles and p-values and are shown to yield very sharp approximations to the exact distribution, exhibiting a clear asymptotic behavior not only for increasing sample sizes but also, and opposite to common asymptotic distributions, for increasing numbers of subsets of variables and increasing numbers of variables in each subset, which amounts to be asymptotic also for the overall number of variables involved. These near-exact distributions are built on a factorization of the characteristic function of the negative logarithm of the likelihood ratio statistic and besides having the above desirable properties they also exhibit a very good performance, that is, a very good proximity to the exact distribution, for very small samples, with this proximity even improving as the overall number of variables involved increases.

Keywords

Beta random variables, Characteristic function, Exact distribution, Likelihood ratio test, Near-exact distributions.

Acknowledgements

This research is partially supported by the Portuguese Fundação para a Ciência e Tecnologia, through Centro de Matemática e Aplicações (CMA-FCT/UNL), project UID/MAT/00297/2019 and by the South-Africa National Research Foundation, through the South African Research Chairs Initiative grant number 71199 (Research Chair grant in Computational and Methodological Statistics – Department of Statistics, University of Pretoria).

References

[1] Liang, Y., D. von Rosen and T. von Rosen (2015). *Testing in multivariate Normal models with block circular covariance structures.* Research Report 2015:2, Department of Statistics, Stockholm University.

Parameter estimation for skew elliptical distributions

Tõnu Kollo, Meelis Käärik, and Anne Selart

University of Tartu, Estonia

Abstract

In the monograph [1] results on skew-normal ja related distributions are summarized. In the following we shall concentrate to the multivariate skew-normal and skew t-distributions. Both distributions have become important in data modelling as well as in building copula models in data analysis (see [2], for example). Estimation creates problems even in the simplest case of skew-normal distribution as explicit expressions of the maximum likelihood estimates are missing. The method of moments can applied but simulation experiments show that convergence to the real values of parameters can be slow. In 20 years since skew elliptical distributions were introduced asymptotic behaviour and confidence regions of estimators have not got much attention. In [3] asymptotic normality for parameter estimators of skew-normal distribution have been established and convergence to the asymptotic distributions was examined by simulation. As a data model, skew normal distribution is not favourable in many applications because of lack of the property of tail dependence. From that point of view multivariate t-distribution is preferred as the tail dependence can be taken into account. Study of the asymptotic behaviour of parameter estimates for skew t-ditribution is a topic of the ongoing project.

Keywords

Asymptotic normality, Skew-normal distribution, Skew t-distribution.

Acknowledgements

This research is supported by the project IUT34-5.

References

[1] Azzalini, A. and A. Capitanio (2014). *The Skew-Normal and Related Families.* Cambridge University Press.

[2] Cherubini, U., E. Luciano and W. Vecchiato (2004). *Copula Methods in Finance.* Wiley

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Case-deletion diagnostics for linear mixed models

Jianxin Pan[1], Yu Fei[2], and Peter Foster[1]

[1] The University of Manchester, UK
[2] Yunnan University of Finance and Economics, China

Abstract

Statistical diagnostics for linear mixed models is always challenging because of the complexity of the models when considering detection of outliers and identification of influential subjects/observations [1]. Little work was done in the literature and lots of efforts have to be made for the research topics [2, 3]. Based on the Q-function, the conditional expectation of the logarithm of the joint-likelihood between responses and random effects, we propose a case-deletion approach to identify influential subjects and influential observations in linear mixed models. The models considered here are very broad in the sense that any covariance structures can be specified in the covariance matrices of the random effects and random errors. Analytically explicit forms of diagnostic measures for the fixed effects and variance components are provided. Comparisons with existing methods, including likelihood-based case-deletion and local influence methods, are made. Numerical results, including real data analysis and simulation studies, are presented for both illustration and comparison.

Keywords

Covariance structures, Generalized Cook distance, Influence analysis, Q-fun-ction.

References

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[2] Beckman, R. J., C. J. Nachtsheim and R. D. Cook (1987). Diagnostics for mixed-model analysis of variance. *Technometrics 29*, 413-426.
[3] Christensen, R., L. M. Pearson and W. Johson (1992). Case-deletion diagnostics for mixed-models. *Technometrics 34*, 38-45.

Holonomic gradient method for multivariate distribution theory

Akimichi Takemura

Shiga University, Hikone, Japan

Abstract

In 2011 we developed a new methodology "holonomic gradient method" (HGM, [3]), which is useful for evaluation of probabilities and normalizing constants of multivariate probability distributions. Since then we have applied HGM to various problems, including distribution of roots of Wishart matrices([1],[2]), orthant probabilities and some distributional problems related to wireless communication([4]). In this talk we give an introduction of HGM and present applications of the method for studying multivariate distribution theory.

Keywords

Algebraic statistics, Gröbner basis, Ordinary differential equation.

Acknowledgements

This research is partially supported by the JSPS Grant-in-Aid for Scientific Research No. 18H04092.

References

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[2] Hashiguchi, H., N. Takayama and A. Takemura (2018). Distribution of the ratio of two Wishart matrices and cumulative probability evaluation by the holonomic gradient method. *Journal of Multivariate Analysis 165*, 270–278.

[3] Nakayama, H., K. Nishiyama, M. Noro, K. Ohara, T. Sei, N. Takayama and A.Takemura (2011). Holonomic gradient descent and its application to the Fisher-Bingham integral. *Advances in Applied Mathematics 47*, 639–658.

[4] Siriteanu, C., A. Takemura, S. Kuriki, H. Shin and C. Koutschan (2015). MIMO Zero-Forcing performance evaluation using the holonomic gradient method. *IEEE Transactions on Wireless Communications 14*, 2322–2335.

Part V

Contributed Speakers – Abstracts

Semantic description of linear mixed model analysis

Hanna Ćwiek-Kupczyńska[1], Katrzyna Filipiak[2], Augustyn Markiewicz[3], Philippe Rocca-Serra[4], Alejandra N. Gonzalez-Beltran[4], Susanna-Assunta Sansone[4], Emilie J. Millet[5], Fred van Eeuwijk[5], Agnieszka Ławrynowicz[6], and Paweł Krajewski[1]

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[2] Institute of Mathematics, Poznań University of Technology, Poland
[3] Department of Mathematical and Statistical Methods, Poznań University of Life Sciences, Poznań, Poland
[4] Oxford e-Research Center, University of Oxford, UK
[5] Biometris, Wageningen University and Research Centre, The Netherlands
[6] Faculty of Computing, Poznań University of Technology, Poland

Abstract

In this talk, we discuss the application of results of linear mixed model (LMM) analysis for the purpose of experimental data management.
Intensive production of data in many research domains requires that the obtained results are efficiently processed and managed. As data reuse is anticipated, especially in experimental life sciences, it becomes increasingly important to share research datasets in public repositories. The availability of comprehensive statistical data summaries can be useful for efficient exploration of those data resources.
To improve data management practices, FAIR data principles [1] have been proposed: scientific data should be findable, accessible, interoperable and reusable. Semantic web technologies [2] provide methods to exchange data in a machine readable and interoperable way. The application of public ontologies to classify individual pieces of information allows to explicitly define their meaning and context, and to make them available as Linked Data [3].
In the present work, we describe a semantic model for the statistical analysis of datasets by linear mixed models [4]. We use ontologies, in particular Statistics Ontology (STATO) [5] to annotate the conclusions from the LMM analysis. We demonstrate the use of our approach for automated processing of datasets obtained from plant phenotyping experiments.

Keywords

experimental data analysis, linear mixed model, FAIR data, scientific data interoperability, Statistics Ontology, semantic web, Linked Data

Acknowledgements

This research is partially supported by the National Science Centre (NCN), project no. 2016/21/N/ST6/02358.

References

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[2] Berners-Lee, T., Hendler, J. (2001). Publishing on the semantic web. *Nature* 26;410(6832):1023-4.
[3] Linked Data: https://www.w3.org/standards/semanticweb/data
[4] Ćwiek-Kupczyńska, H., Filipiak, K., Markiewicz, A., Rocca-Serra, P., Gonzalez-Beltran, A.N., Sansone, S-A., Millet, E.J., van Eeuwijk, F., Ławrynowicz, A., Krajewski, P. (2019). A semantic concept schema of the linear mixed model of experimental observations. Submitted.
[5] STATO 2012, statistics ontology: http://stato-ontology.org

Some comments on maximum likelihood estimation under various statistical models

Katarzyna Filipiak

Poznań University of Technology, Poland

Abstract

The aim of this talk is discuss relations between the conditions of the existence of maximum likelihood estimators of covariance structures:

- in explicit form (cf. Szatrowski, 1980),
- as projections (cf. Fuglede and Jensen, 2013).

This topic is inspired by the invariance property of maximum likelihood estimators (Zehna, 1966).

Keywords

Maximum likelihood estimator, Multivariate model, Jordan algebra, Projection.

References

[1] Fuglede, B. and S.T. Jensen (2013). Positive projections of symmetric matrices and Jordan algebras. *Expo. Math. 31*, 295–303.

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On properties of Lee's bivariate spatial association measure

Jan Hauke and Tomasz Kossowski

Adam Mickiewicz University, Poznań, Poland

Abstract

Sang-Il Lee ([2], [3]) developed and analysed a new bivariate spatial association measure (cited by many authors who discussed the measure from theoretical as well as practical point of view). He noted that a bivariate spatial association measure should be a composite of three elements: univariate spatial associations of two variables and their point-to-point association in a certain form. The idea led him to construction the bivariate measure (called L) connected with decomposition of Moran's I. The L statistics integrates Pearson's r and Moran's I and heavily depends on (introduced by Sang-Il Lee) a spatial smoothing scalar (SSS). In the paper we examine some results obtained by Sang-Il Lee dropping (changing) some theoretical assumption used in the definition (decomposition) of L.

Keywords

Pearson's r, Spatial autocorrelation, Moran's I, Bivariate spatial association measure.

References

[1] Anselin, L. (2019). A Local Indicator of Multivariate Spatial Association: Extending Geary's C. *Geographical Analysis 51*, 133–150.

[2] Lee, SI. (2001). Developing a bivariate spatial association measure: an integration of Pearson's r and Moran's I. *Journal of Geographical Systems 3*, 369–385.

[3] Lee, SI. (2004). A generalized significance testing method for global measures of spatial association: an extension of the Mantel test. *Environment and Planning A 36*, 1687–1703.

[4] Lee, SI. (2009). A generalized randomization approach to local measures of spatial association. *Geographical Analysis 41*, 221–248.

[5] Lee, SI. (2017). Correlation and Spatial Autocorrelation. In: S. Shekhar, H. Xiong, X. Zhou. (Eds), *Encyclopedia of GIS*, 14–92, Springer.

Maximum likelihood estimation in applications

Malwina Janiszewska, Anna Szczepańska-Álvarez, and Emilia Zawieja

Poznań University of Life Sciences, Poland

Abstract

The aim of the presentation is the analysis of the experiment where some characteristics are observed in the time points and others do not change in the time. The relations between these two types of features is interested. We assume the dispersion matrix as the block matrix in two cases: without the structure and with Kronecker product structure. We determine the maximum likelihood estimators. The results of two cases are compared and illustrated by the example.

Keywords

Dispersion matrix, Kronecker product, Maximum likelihood estimation.

References

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Testing hypotheses about banded Toeplitz covariance structure under the high-dimensional models

Mateusz John

Poznań University of Technology, Poland

Abstract

The main goal of the presentation is to propose the test for banded Toeplitz covariance structure and verify its properties. The tests based on likelihood ratio and Rao score with the maximum likelihood estimators replaced by asymptotic estimator of banded Toeplitz matrix ([1]) or shrinkage estimator of banded Toeplitz matrix ([2]) are indicated.
It is known that likelihood ratio test as well as Rao score test have asymptotic chi-square distribution. Thus, the aim will be to verify if the proposed modified likelihood ratio and Rao score also follow this distribution.

Keywords

Banded Toeplitz matrix, Likelihood ratio test, Rao score test, Asymptotic estimator, Shrinkage estimator.

References

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[2] John, M. and A. Mieldzioc (2019). The comparison of the estimators of banded Toeplitz covariance structure under the high-dimensional multivariate model, *Comm. Statist. Simulation Comput.* Accepted.

On a simplified approach to estimation in experiments with orthogonal block structure

Radosław Kala

Poznań University of Life Sciences, Poland

Abstract

The experiments with the orthogonal block structure form a wide class of designs having, under assumption of full randomization, the dispersion matrix of a special spectral form with unknown variance components. In the paper it is shown how the known estimation procedures of both the treatment parameters and variance components can be simplified. The approach proposed is direct, quite general and mainly uses the technique of orthogonal projection.

Keywords

Block designs, Row and column designs, Nested block designs, Orthogonal projectors, Stratum submodels.

References

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[2] Houtman, A.M. and T.P. Speed (1983). Balance in designed experiments with orthogonal block structure. *Ann. Math. Statist. 11,* 1069-1085.

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[5] Nelder, J.A. (1968). The combination of information in generally balanced designs. *J Roy. Statist. Soc. Ser B 30,* 303-311.

Growth curve model with compound symmetry structure

Veronika Kopčová, Ivan Žežula, and Daniel Klein

P. J. Šafárik University, Košice, Slovakia

Abstract

Testing in the growth curve model (GCM) will be considered. As the number of variance-covariance parameters grows very quickly with the dimension, special variance structures with reduced number of parameters are intensively studied in recent years. One of the simplest structures of the variance matrix is the compound symmetry structure which is characterized by just two parameters regardless of the dimension. Khatri developed likelihood ratio test procedure for testing presence of this structure in GCM in 1973. Recently it was proposed other alternative based on the spectral decomposition. We compare these tests and we also derived simultaneous test of specific mean of the model and compound symmetry structure.

Keywords

Growth curve model, Compound symmetry structure, Simultaneous test of mean and compound symmetry structure.

Acknowledgements

This work was supported by the Slovak Research and Development Agency under the contract no. APVV-17-0568 and by grant VEGA 1/0311/18.

Simultaneous testing hypotheses in models with blocked compound-symmetric covariance structure

Roman Zmyślony and Arkadiusz Kozioł

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Abstract

In the presentation will be considered simultaneous testing hypotheses about structure of mean vector and covariance matrix in models with some special covariance structure, so-called blocked compound-symmetric (BCS) covariance structure, for $m-$variate observations over u levels of some factor on each of n individual under the assumption of multivariate normality. Using framework of ratio of positive and negative parts of best unbiased estimators test statistic for simultaneous test is obtained and can be proved that under null hypothesis the statistic has exact F distribution. Simulation study is conducted to show strong and weak sides of considered test which is compared with two other F tests for testing single hypotheses about mean and covariance matrix.

Keywords

Best unbiased estimator, Testing hypotheses, Structure of covariance matrices, Structure of mean vector, Positive and negative part of estimator, Block compound symmetric covariance structure, Double multivariate data.

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On sufficiency of quadratically sufficient statistics in possibly mixed model

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Abstract

The notion of quadratic sufficiency was introduced and characterized in [1] in the context of fixed linear model. It was proved there that under normality quadratically sufficient statistic is also sufficient. In the paper it is studied the problem of estimation in possibly misspecified model; i.e. when some effects assumed to be fixed are random. It is shown that quadratically sufficient statistic under fixed model it is sufficient under respective mixed linear normal model. The results are applied to provide data reduction in factorial experiment.

Keywords

Linear sufficiency, Quadratic sufficiency, Mixed model.

References

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The comparison of the estimators of banded Toeplitz covariance structure

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[2] Poznań University of Life Sciences, Poland

Abstract

We consider the model of experiment in which several characteristics are measured or the measurements are repeated in time. We assume that the covariance matrix of the characteristics/time points is unknown, but has the banded Toeplitz structure. The problem of estimating such a covariance structure was considered by e.g. Cui et al. (2016) [1], Filipiak et al. (2018) [2].
We discuss two methods that aim to deal with singularity and numerical ill-conditioning of the estimators of covariance matrix, especially under high-dimensional regime.
The method of estimation of banded Toeplitz covariance matrix based on a shrinking, described by Ledoit and Wolf (2004) [3] for unstructured covariance matrices is proposed. The estimators obtained with the use of the proposed method are then compared with the estimators obtained by the projection of sample covariance matrix onto the asymptotic cone of the nonnegative definite Toeplitz matrices (Filipiak et al., 2018). For this purpose simulation studies concerning bias and risk are conducted for several sets of parameters.

Keywords

Covariance matrix, Estimation, Projection, Shrinkage, Toeplitz matrix.

Acknowledgements

This research is partially supported by Statutory Activities Number 04/43/DSPB/0101.

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Covariance matrix approximation in doubly multivariate models

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Institute of Plant Genetics, Polish Academy of Sciences, Poznań, Poland

Abstract

Covariance matrix estimation is a challenging problem in multivariate statistics, especially for experiments with small sample size and a number of characteristics and time points. Such models are very often overparameterized and then the choice of particular covariance structure can be useful.
The aim of this talk is to find the best approximation of a positive definite matrix by a matrix from a given set of suitable structures with respect to some discrepancy function. For doubly multivariate data Kronecker product of covariance matrix for characteristics and covariance matrix for time points seems to be the most relevant structure. Moreover, one of those covariance structures can be additionally restricted by for example compound symmetry or autoregression of order one structure. In this talk three discrepancy functions: the Frobenius norm, the entropy loss function and the quadratic loss function will be presented.
Our goals are: approximation of a given matrix by a covariance structure (regularization), comparison of statistical properties of the best approximations (estimators) of the covariance matrix and maximum likelihood estimator and the use of the best approximation to calculate the discrepancies between the hypotheses in the power test studies.

Keywords

Covariance structure, Compound symmetry, Autoregression, Regularization.

References

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Comments on the recursive formula on spectral moments of Wishart matrix and on the Rao score test statistics under model with BCS structured covariance matrix

Jolanta Pielaszkiewicz

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Abstract

Presentation consists of two distinct parts, although clearly in whole talk the interest in covariance matrix and Wishart matrix as such is expressed.
Firstly, the new comments on results regarding general recursive formula for $\mathrm{E}[\prod_{i=0}^{k}\mathrm{tr}\{W^{m_i}\}]$, where $W \sim \mathcal{W}_p(I,n)$ denotes a real Wishart matrix will be given.
The second part of the talk focus on the distribution of Rao score test statistics under the multivariate model with block compound symmetry covariance structure. Here, presentation includes work in progress that after improvements aim to be published in joint paper with K. Filipiak and D. Klein that are authors of [2].

Keywords

Wishart matrix, Spectral moments, Recursive formula, Rao score test, Block compound symmetry structure.

References

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Optimal designs in multiple group random coefficient regression models

Maryna Prus

Otto von Guericke University, Magdeburg, Germany

Abstract

Random coefficients regression (RCR) models have been introduced in bio-sciences for selection purposes and are nowadays popular in many fields of statistical applications, for example in medical research and pharmacology. Optimal designs for the estimation of population (fixed) parameters are well discussed in the literature (see e. g. [2]). RCR models with known population (mean) parameters were investigated by [1]. [3] proposed solutions for optimal designs for the prediction of individual random parameters in models with unknowns population mean under assumption of the same treatment for all individuals. This talk presents analytical results for optimal designs for the prediction in multiple group RCR models, where different treatments are allowed for different groups.

Keywords

Mixed models, Multiple group models, Optimal designs, Prediction.

Acknowledgements

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A review of the linear prediction sufficiency in the linear model with new observations

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Abstract

We consider the general linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}+\boldsymbol{\varepsilon}$, denoted as $M = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$, supplemented with the new unobservable random vector $\mathbf{y}_*$, coming from $\mathbf{y}_* = \mathbf{X}_*\boldsymbol{\beta} + \boldsymbol{\varepsilon}_*$, where the covariance matrix of $\mathbf{y}_*$ is known as well as the cross-covariance matrix between $\mathbf{y}_*$ and $\mathbf{y}$. A linear statistic $\mathbf{Fy}$ is called linearly sufficient for $\mathbf{X}_*\boldsymbol{\beta}$ if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased estimator, BLUE, for $\mathbf{X}_*\boldsymbol{\beta}$. The concept of linear sufficiency with respect to a predictable random vector is defined in the corresponding way but considering the best linear unbiased predictor, BLUP, instead of BLUE. In this paper, we consider the linear sufficiency of $\mathbf{Fy}$ with respect to $\mathbf{y}_*$, $\mathbf{X}_*\boldsymbol{\beta}$, and $\boldsymbol{\varepsilon}_*$, when the prediction is based on M. We also apply our results into the linear mixed model.

Keywords

BLUE, BLUP, Linear sufficiency, Linear model with new observations, Linear mixed model, Transformed linear model.

Acknowledgements

This research is joint work with Stephen J. Haslett, Jarkko Isotalo, Radosław Kala and Augustyn Markiewicz.

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The Growth Curve model under high dimensions

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Abstract

The Growth Curve model is a bilinear model useful for studying short balanced time series. In high dimensions the mean parameter space is fixed but the size of the dispersion matrix becomes large, meaning that there are an infinite number of nuisance parameters. Estimators of mean parameters are derived under high dimensional assumptions. The proposed estimator is unbiased and an upper error bound for its dispersion is given.

Keywords

Growth Curve model, Kolmogorov asymptotic, Moment calculations.

MLE of separable covariance matrix structure – simulation study

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[1] Collegium Da Vinci, Poland
[2] Poznań University of Life Sciences, Poland

Abstract

We are interested in the estimation of separable covariance matrix structure where the first matrix is unstructured and the second one is the partitioned matrix. In the considered case the maximum likelihood estimator has not the explicit form and only the set of equations is given. We analyse the convergence and the bias of solution determined by the algorithm and present the simulations studies.

Keywords

Convergence, Bias of estimator, Separable covariance structure, Kronecker product, Maximum likelihood estimation.

References

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Distribution of the squared first antieigenvalue

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[2] University of Rwanda, Rwanda

Abstract

For fifty years ago Karl Gustafson published a series of papers and developed an antieigenvalue theory which has been applied, in a non-statistical manner, to several different areas including, numerical analysis and wavelet analysis, quantum mechanics, finance and optimisation. The first antieigenvector $\mathbf{u}_1$ (actually there are two) is the vector which is the one which is the most "turned" by an action of a positive definite matrix $\mathbf{A}$ with a connected antieigenvalue μ_1 which indeed is the cosine of the maximal "turning" angle given as

$$\mu_1 = \frac{2\sqrt{\lambda_1 \lambda_p}}{\lambda_1 + \lambda_p},$$

where λ_1 is the largest and λ_p is the smallest eigenvalue of $\mathbf{A}$, respectively. Antieigenvalues have been introduced in statistics, for example, as a measures of efficiency of least squares estimators, and when testing for sphericity, see [1, 2, 3]. In this talk we will consider the distribution of the squared first antieigenvalue and discuss the use of it.

Keywords

Eigenvalue, Aniteigenvalue, Probability distribution.

References

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Large deviation probabilities of condition numbers of sample covariance matrices

Martin Singull, Denise Uwamariya, and Xiangfeng Yang

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Abstract

A square matrix is defined as $\mathbf{W}_{p\times p} = \mathbf{X}\mathbf{X}^T/n$ for $2 \leq p \leq n$, where $\mathbf{X}$ is a $p \times n$ random matrix whose entries X_{ij} are i.i.d. with zero mean and unit variance. The aim of this paper is to study the large deviation probabilities of the condition number of $\mathbf{W}$ as $n \to \infty$. Results are obtained (i) when X_{ij} are standard normal and $p = o(n)$, and (ii) when X_{ij} are general and $p = o(n/\ln\ln n)$.

Keywords

Condition number, Wishart matrix, Large deviation.

References

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Computing the exact distribution of selected test statistics in multivariate analysis

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Abstract

As suggested in [1] and [2], the exact null and/or null-distribution of the most common likelihood ratio based test statistics in multivariate analysis can be expressed by their characteristic functions. Typically, such distributions are approximated by the standard asymptotic chi-square approximation. The small sample approximations based on first moments/cumulants include the Edgeworth or Gram-Charlier expansions about the known distribution. In specific situations the more sophisticated approximations can be used, such as the near-exact distributions suggested by Carlos A. Coelho and his co-authors. For most applications, the method based on numerical inversion of the characteristic functions is sufficient. Applicability of such approach is illustrated and compared by computing the exact null and non-null distributions of selected test statistics used in multivariate hypothesis testing, [3].

Keywords

Characteristic function, Numerical inversion, Exact distribution.

Acknowledgements

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Testing means in two-level compound symmetry multivariate data

Ivan Žežula

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Abstract

Many experimental designs, especially in medical sciences, lead to special variance structures in multivariate data. Since the number of observations is rarely large, it is important to take this special structure into account as it substantially reduces the number of estimated parameters.
We consider simple multivariate model

$$\boldsymbol{X} = \boldsymbol{M} + \boldsymbol{E},$$

where $\boldsymbol{M}$ is a location or mean matrix, and $\boldsymbol{E}$ is an error matrix. The variance matrix of $\boldsymbol{E}$ is assumed to be doubly-exchangeable, i.e. block-wise compound symmetry. We develop test procedures for all basic tests of location needed in various applications.

Keywords

Multivariate linear model, Location testing, Special variance structures, Two-level compound symmetry.

Acknowledgements

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Part VI

MMLM Research Meetings

MMLM meetings - a little bit of history

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The International Conference on Multivariate and Mixed Linear Models, MMLM 2019, will sum up the research meetings on

- *Multivariate Linear Models and Designs of Experiments*,
- *Sufficient and Optimal Statistical Procedures in Mixed Linear Model*,
- *Planning and Analysis of Tensor-Experiments*,
- *Neighbour Designs and Crossover Designs*,
- *Mixed and Multivariate Models*,
- *Multivariate Linear Models*, and
- *Multivariate and Mixed Linear Models*,

which have been held in Będlewo since 2009, usually twice per year.

The research meetings were inspired by Professor Dietrich von Rosen (Swedish University of Agriculture, Uppsala, Sweden), to keep Polish-Swedish collaboration in Multivariate Models. He proposed to invite Professor Roman Zmyślony (University of Zielona Góra, Poland), working in Mixed Models area, and then the first research group meeting on *Multivariate Linear Models and Designs of Experiments* was held in Będlewo in November 2009. Since the fourth research meeting in the fall of 2013, one of the main topics has been the application of tensor operators to statistical modeling of highly complex data. The main topics, initiated by Dietrich von Rosen in 2013, have been optimal estimation and hypothesis testing on covariance structures.
It is worth noting that the first Polish-Swedish research meeting became international, and during all these years 49 participants from 11 countries have attended the meetings. In the research meetings about 11 Ph.D. students have actively participated and started collaboration with more senior colleagues. As the result of the meetings, over 50 peer-reviewed articles and one book has been published or submitted to international scientific journals.

In the following we give a list of all the meetings together with the number of participants with respect to their country affiliation and a list of articles inspired by the series of research meetings.

List of all MMLM meetings

	Date	Meeting title	Affiliation country and the number of participants
1.	11-16.11.2009	Multivariate linear models and designs of experiments	Poland 5 Sweden 1
2.	08-13.11.2010	Multivariate linear models and designs of experiments	Poland 5 Portugal 3
3.	11-17.11.2012	Sufficient and optimal statistical procedures in mixed linear model	Poland 5 Portugal 2 Germany 1 Slovakia 1
4.	22-26.04.2013	Planning and Analysis of Tensor-Experiments	Poland 3 Sweden 3 Slovakia 2 USA 1
5.	20-24.05.2013	Neighbour designs and crossover designs	Poland 3 France 1 Germany 1 UK 1
6.	21-25.11.2013	Mixed and Multivariate Models	Poland 7 Slovakia 2 Finland 1 Germany 1 Portugal 1 Sweden 1 USA 1
7.	24-28.03.2014	Multivariate linear models	Poland 6 Slovakia 2 Sweden 1 USA 1
8.	08-15.11.2014	Multivariate linear models	Poland 7 Sweden 2 Estonia 1 Finland 1 Portugal 1 Slovakia 1 USA 1
9.	07-13.03.2015	Multivariate linear models	Poland 8 Slovakia 2 Finland 1 Germany 1 Sweden 1 USA 1
10.	08-14.11.2015	Multivariate and Mixed Linear Models	Poland 8 Portugal 3 Slovakia 2

Continued on next page

Table 1 – *Continued from previous page*

Date	Meeting title	Affiliation country and the number of participants	
		Finland	1
		Sweden	1
11. 14-18.03.2016	Multivariate and Mixed Linear Models	Poland	7
		Portugal	5
		Slovakia	2
		Finland	1
		Sweden	1
		USA	1
12. 07-11.11.2016	Multivariate and Mixed Linear Models	Poland	10
		Slovakia	3
		Finland	1
		Portugal	1
		Sweden	1
		UK	1
13. 13-17.03.2017	Multivariate and Mixed Linear Models	Poland	13
		Slovakia	2
		Finland	1
		Portugal	1
		Sweden	1
14. 06-10.11.2017	Multivariate and Mixed Linear Models	Poland	11
		Slovakia	2
		Sweden	2
		Finland	1
		Iran	1
15. 19-23.03.2018	Multivariate and Mixed Linear Models	Poland	9
		Sweden	4
		Slovakia	2
		Finland	1
		Portugal	1
16. 11-17.11.2018	Multivariate and Mixed Linear Models	Poland	15
		Slovakia	2
		Finland	1
		Sweden	1

List of publications: 2009 – 2018

Books:

1. von Rosen, D. (2018). *Bilinear Regression Analysis: An Introduction*. Springer (ISBN 978-3-319-78784-8).

Articles:

2010:

1. Nahtman, T., D. von Rosen (2010). On a new class of singular nonsymmetric matrices with nonnegative integer spectra. In: *Matrix Methods: Theory, Algorithms and Applications: Dedicated to the Memory of Gene Golub*, V. Olshevsky, E. Tyrtyshnikov (Eds). World Scientific Publishing, 140–165.
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2011:

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2012:

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