A review of the linear prediction sufficiency in the linear model with new observations

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Abstract

We consider the general linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, denoted as $M = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$, supplemented with the new unobservable random vector $\mathbf{y}_*$, coming from $\mathbf{y}_* = \mathbf{X}_*\boldsymbol{\beta} + \boldsymbol{\varepsilon}_*$, where the covariance matrix of $\mathbf{y}_*$ is known as well as the cross-covariance matrix between $\mathbf{y}_*$ and $\mathbf{y}$. A linear statistic $\mathbf{Fy}$ is called linearly sufficient for $\mathbf{X}_*\boldsymbol{\beta}$ if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased estimator, BLUE, for $\mathbf{X}_*\boldsymbol{\beta}$. The concept of linear sufficiency with respect to a predictable random vector is defined in the corresponding way but considering the best linear unbiased predictor, BLUP, instead of BLUE. In this paper, we consider the linear sufficiency of $\mathbf{Fy}$ with respect to $\mathbf{y}_*$, $\mathbf{X}_*\boldsymbol{\beta}$, and $\boldsymbol{\varepsilon}_*$, when the prediction is based on M. We also apply our results into the linear mixed model.

Keywords

BLUE, BLUP, Linear sufficiency, Linear model with new observations, Linear mixed model, Transformed linear model.

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