Large deviation probabilities of condition numbers of sample covariance matrices

Martin Singull[1] **Denise Uwamariya**[2]
and Xiangfeng Yang[3]

[1,2,3] *Linköping University, Sweden*

Abstract

A square matrix is defined as $\mathbf{W}_{p\times p} = \mathbf{X}\mathbf{X}^T/n$ for $2 \leq p \leq n$, where $\mathbf{X}$ is a $p \times n$ random matrix whose entries X_{ij} are i.i.d. with zero mean and unit variance. The aim of this paper is to study the large deviation probabilities of the condition number of $\mathbf{W}$ as $n \to \infty$. Results are obtained (i) when X_{ij} are standard normal and $p = o(n)$, and (ii) when X_{ij} are general and $p = o(n/\ln\ln n)$.

Keywords

Condition number, Wishart matrix, Large deviation.

References

[1] Anderson, W. and M. Wells (2009). The exact distribution of the condition number of a Gaussian matrix. *SIAM J. Matrix Anal. Appl. 31,* 1125–1130.

[2] Bai, Z., J. Silverstein and Y. Yin (1988). A note on the largest eigenvalue of a large-dimensional sample covariance matrix. *J. Multivariate Anal. 26,* 166–168.

[3] Chen, Z. and J. Dongarra (2005). Condition numbers of Gaussian random matrices. *SIAM J. Matrix Anal. Appl. 27,* 603–620.

[4] Dembo, A. and O. Zeitouni (2010). *Large deviations techniques and applications.* (Corrected reprint of the 2nd ed). Springer-Verlag.

[5] Edelman, A. (1988). Eigenvalues and condition numbers of random matrices. *SIAM J. Matrix Anal. Appl. 9,* 543–560.

[6] Edelman, A. and B. Sutton (2005). Tails of condition number distributions. *SIAM J. Matrix Anal. Appl. 27,* 547–560.

[7] Fey, A., R. van der Hofstad and M. Klok (2008). Large deviations for eigenvalues of sample covariance matrices, with applications to mobile communication systems. *Adv. in Appl. Probab. 40,* 1048–1071.

[8] Gustafson, K. (2012). *Antieigenvalue analysis.* World Scientific Publishing Co.

[9] James, A. (1964). Distributions of matrix variates and latent roots derived from normal samples. *Ann. Math. Statist. 35,* 475–501.

[10] Jiang, T. and D. Li (2015). Approximation of rectangular beta-Laguerre ensembles and large deviations. *J. Theoret. Probab. 28,* 804–847.

[11] Kevei, P. (2010). A note on asymptotics of linear combinations of iid random variables. *Period. Math. Hungar. 60,* 25–36.

[12] Muirhead, R. (1982). *Aspects of multivariate statistical theory.* John Wiley & Sons, Inc.

[13] Rogers, C. (1963). Covering a sphere with spheres. *Mathematika 10,* 157–164.

[14] Silverstein, J. (1985). The smallest eigenvalue of a large-dimensional Wishart matrix. *Ann. Probab. 13,* 1364–1368.

[15] Solomyak, B. (1995). On the random series $\sum \pm\lambda^n$ (an Erdős problem) *Ann. of Math. 142,* 611–625.

[16] Srivastava, M. and C. Khatri (1979). *An introduction to multivariate statistics.* North-Holland.