SELF-SIMILAR SOLUTIONS OF KINETIC-TYPE EQUATIONS

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For a time dependent family of probability measures $(\rho_t)_{t\geq 0}$ we are interested in the solution of the following Cauchy problem

$$\begin{aligned} \frac{\partial}{\partial t}\phi(t,\xi) + \phi(t,\xi) &= \widehat{Q}(\phi(t,\cdot),\dots,\phi(t,\cdot))(\xi), \quad t>0,\ \xi\in\mathbf{R} \\ \phi(0,\xi) &= \phi_0(\xi), \quad \xi\in\mathbf{R}, \end{aligned}$$

where $\widehat{Q}$ is a smoothing transform and ϕ_t is the Fourier–Stieltjes transform of ρ_t. Assuming that the initial measure ρ_0 belongs to the domain of attraction of a stable law, we describe asymptotic properties of ρ_t, as $t\to\infty$.

We consider the boundary regime when the standard normalization leads to a degenerate limit and find an appropriate scaling ensuring a non-degenerate self-similar limit. As we will see, the right normalization involves a subexponential term and the limit is a fixed point of some smoothing transform. Our approach is based on a probabilistic representation of probability measures $(\rho_t)_{t\geq 0}$ that refines the corresponding construction proposed in [1].

This talk is based on joint work with Dariusz Buraczewski and Alexander Marynych (see [2])

References

[1] Bassetti, F. and Ladelli, L., *Self-similar solutions in one-dimensional kinetic models: A probabilistic view*, Ann. Appl. Probab., 22(5), 1928–1961, (2012).

[2] K. Bogus, D. Buraczewski, A. Marynych, *Self-similar solutions of kinetic-type equations: the critical case.*, to appear in Stochastic Processes and their Applications, p. 18 (2019),