RENEWAL APPROXIMATION IN THE ONLINE INCREASING SUBSEQUENCE PROBLEM

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Suppose a sequence of independent marks is drawn from a continuous distribution and observed at times of the unit Poisson process. Each mark can be selected or rejected, with decision becoming at the moment of observation immediately final. The objective is to maximise the expected length of increasing subsequence which can be selected over horizon t by a nonanticipating online strategy.

Samuels and Steele [4] introduced the problem and proved the principal large-t asymptotics $v(t) \sim \sqrt{2t}$, achieved by a simple strategy with constant acceptance window $\sqrt{2/t}$. Bruss and Delbaen [1, 2] used concavity and delicate martingale arguments to obtain finer estimates and a CLT for the length L_t under more complex optimal policy with state-dependent window.

In this talk we refine known results by deriving the asymptotic expansion

$$v(t) = \sqrt{2t} - \frac{1}{12}\log t + c - \frac{\sqrt{2}}{144\sqrt{t}} + O(t^{-1}),\ \ t \to \infty,$$

with some (yet unknown) constant c. The expansion with $O(1)$ remainder is proved by comparing solutions to the optimality equation

$$v'(t) = \int_0^1 (v(t(1-x)) + 1 - v(t))_+ \, dx$$

with suitable test functions. Proving convergence of the $O(1)$ term and expanding beyond it is done by first showing that on the scale $z = \sqrt{t}$ the optimisation problem can be represented via a controlled piecewise-deterministic Markov process Z, which is decreasing and asymptotically homogeneous in the sense of [3]. Analysis of the potential measure of Z allows us to explain the logarithmic term in the asymptotic expansion, to obtain a similar expansion for $\mathrm{Var}(L_t)$ and to prove the CLT by elementary comparison with an alternating renewal process.

References

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