

# WEAK OPTIMAL TRANSPORT AND APPLICATIONS TO CAFFARELLI CONTRACTION THEOREM

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The talk will deal with a variant of the optimal transport problem, introduced in a joint work with C. Roberto, P-M Samson and P. Tetali [3], where elementary mass transports are penalized through their barycenters. The talk will in particular focus on a recent result obtained in collaboration with N. Juillet [2] describing optimal transport plans for the quadratic barycentric cost. A direct corollary of this result gives a new necessary and sufficient condition for the Brenier map to be 1-Lipschitz. Finally we will present a recent work in collaboration with M. Fathi and M. Prodhomme [1], where this contractivity criterion is used to give a new proof of the celebrated Caffarelli contraction theorem, telling that any probability measure having a log-concave density with respect to the standard Gaussian measure is a contraction of it.

## REFERENCES

- [1] Fathi, Max; Gozlan, Nathael; Prodhomme, Maxime, *A proof of the Caffarelli contraction theorem via entropic regularization*, preprint (2019). Available at <https://arxiv.org/abs/1904.06053>.
- [2] Gozlan, Nathael; Juillet, Nicolas, *On a mixture of Brenier and Strassen Theorems*, preprint (2018). Available at <https://arxiv.org/abs/1808.02681>.
- [3] Gozlan, Nathael; Roberto, Cyril; Samson, Paul-Marie; Tetali, Prasad, *Kantorovich duality for general costs and applications*, *Journal of Functional Analysis* 273 (2017), no. 11, 3327–3405.