ON SEMIGROUPS ASSOCIATED WITH THE DUNKL OPERATORS

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Dunkl theory is a generalization of Fourier analysis and special function theory related to root systems and reflections groups. The Dunkl operators T_j, which were introduced by C. F. Dunkl in 1989, are deformations of directional derivatives by difference operators related to the reflection group. We shall discuss the semigroups associated with T_j i.e. by the Dunkl Laplacian $\Delta = \sum_{j=1}^{N} T_j^2$, which will be our starting point. We will study the behaviour of Dunkl heat kernel $h_t(\mathbf{x}, \mathbf{y})$ and provide the estimates in a spirit of analysis on spaces of homogeneous type, expressed in term of Euclidean metric and the orbit distance. Then, we will see how to use the estimates of $h_t(\mathbf{x}, \mathbf{y})$ to prove the boundedness of some operators and to prove the estimates for the kernels of operators associated with the other semigroups i.e. the Dunkl Poisson semigroup generated by $\sqrt{\Delta}$. In the one-dimensional case and in the product case the kernels of some operators can be expressed explicitly in terms of classical special functions. In the general case considered in this talk, no such information is available. We will discuss the main differences between the classical and the rational Dunkl setting and provide some methods how to omit difficulties which are caused by the lack of knowledge on some basic objects in the Dunkl theory (i.e. the boundedness of the Dunkl translations on $L^p(dw)$ for $p \neq 2$).

This talk is based on the joint articles with J-Ph. Anker and J. Dziubański.