SUBCRITICAL GALTON-WATSON BRANCHING PROCESSES WITH IMMIGRATION IN RANDOM ENVIRONMENT

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The Galton–Watson process with immigration in random environment is defined as follows. Let $X_0 = 0$, and

$$X_{n+1} = \sum_{i=1}^{X_n} A_i^{(n+1)} + B_{n+1}, \quad n \geq 0,$$

where, conditioned on the environment $\mathcal{E} = (\epsilon_1, \epsilon_2, \ldots)$ the number of offsprings and immigrants $\{A_i^{(n)}, B_n : i = 1, 2, \ldots, n = 1, 2, \ldots\}$ are independent random variables, and for n fixed $(A_i^{(n)})_{i=1,2,\ldots}$ are iid. Here ϵ_i stands for the random environment in generation i, the variable $A_i^{(n+1)}$ represents the number of offsprings of the ith element in the nth generation, and B_n is the number of immigrants in the nth generation.

We assume that the process is subcritical, i.e. $\mathbf{E} \log \mathbf{E}[A|\epsilon] < 0$, and Cramér's condition holds:

$$\mathbf{E}(\mathbf{E}[A|\epsilon])^\kappa = 1 \quad \text{for some } \kappa > 0.$$

Using Goldie's implicit renewal theory, we determine the tail behavior of the stationary solution. We prove the following.

Theorem 1. *Under the preceding assumptions the process has a unique stationary distribution μ for which*

$$\mu((x, \infty)) \sim C x^{-\kappa} \quad \textit{as } x \to \infty,$$

for some constant $C \geq 0$. Moreover, $C > 0$ for $\kappa \geq 1$.

We note that the original motivation comes from Kesten, Kozlov, and Spitzer [1], where a random walk in random environment model reduces to a special Galton–Watson process with immigration in random environment.

References

[1] Kesten, H.; Kozlov, M. V.; Spitzer, F. A limit law for random walk in a random environment. *Compositio Math.* 30 (1975), 145–168.