HARDY–LITTLEWOOD MAXIMAL OPERATORS IN NON-DOUBLING SETTING

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It is well known that for any metric measure space (X, ρ, μ) with μ satisfying the so-called doubling condition, the associated Hardy–Littlewood maximal operators, centered $\mathcal{M}^c$ and non-centered $\mathcal{M}$, are bounded on L^p, $p \in (1, \infty]$, and from L^1 to $L^{1,\infty}$.

The situation is different if non-doubling spaces are considered instead. For example, Sjögren [1] showed that $\mathcal{M}$ is not bounded from L^1 to $L^{1,\infty}$ in the case of $\mathbb{R}^2$ with the Gaussian measure $d\mu(x, y) = e^{-(x^2+y^2)/2}\, dxdy$.

Our aim is to introduce a class of non-doubling spaces for which the associated maximal operators have very specific mapping properties. In particular, for any fixed $p_0 \in (1, \infty)$ we construct a space $\mathfrak{X}$ such that $\mathcal{M}^c$ and $\mathcal{M}$ are bounded on $L^p(\mathfrak{X})$ if and only if $p \geq p_0$. Furthermore, we prove the following theorem.

Theorem 1. *Let $1 < p_1 < p_2 < \infty$. Then there exists a non-doubling metric measure space for which the ranges of boundedness on L^p for the associated operators $\mathcal{M}^c$ and $\mathcal{M}$ are of the form $[p_1, \infty]$ and $[p_2, \infty]$, respectively.*

References

[1] P. Sjögren, *A remark on the maximal function for measures in R^n*, Amer. J. Math. **105** (1983), 1231–1233