SINGULAR STOCHASTIC INTEGRAL OPERATORS

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Singular integral operators play a prominent role in harmonic analysis. By replacing the integration with respect to some measure by integration with respect to Brownian motion, one obtains a stochastic singular integral of the form

$$S_K G(t) := \int_0^\infty K(t,s)G(s)\,\mathrm{d}W_H(s), \qquad t \in \mathbb{R}_+,$$

which appears naturally in questions related to stochastic maximal regularity. Here G is an adapted process, W_H is a cylindrical Brownian motion and K is allowed be singular. In this talk I will introduce Calderón–Zygmund theory for such singular stochastic integrals with operator-valued kernel K. I will first discuss L^p-extrapolation under a Hörmander condition on the kernel. Afterwards I will treat sparse domination and sharp weighted bounds under a Dini condition on the kernel, leading to a stochastic analog of the solution to the A_2-conjecture.

The developed theory implies p-independence and weighted bounds for stochastic maximal L^p-regularity both in the complex and real interpolation scale. This leads to mixed $L^p(L^q)$-theory for several stochastic partial differential equations, of which I will give a few examples.