ON DELOCALIZATION OF EIGENVECTORS OF RANDOM NON-HERMITIAN MATRICES

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We study delocalization properties of null vectors and eigenvectors of matrices with i.i.d. subgaussian entries. Such properties describe quantitatively how "flat" is a vector or how close is its distribution to the uniform distribution on the unit sphere. Here we address the so called *no-gaps delocalization* (see [2]). A vector possessing this property has no "gaps" in its coordinate profile and every subset of its components carries a non-negligible mass. Our main result can be formulated as follows:

Theorem 1. *Let A be an $n \times n$ random matrix with i.i.d real subgaussian entries of zero mean and unit variance. We show that with probability at least $1 - e^{-\log^2 n}$*

$$\min_{I \subset [n],\, |I|=m} \|\mathbf{v}_I\| \geq \frac{m^{3/2}}{n^{3/2} \log^C n} \|\mathbf{v}\|$$

for any real eigenvector $\mathbf{v}$ and any $m \in [\log^C n, n]$, where $\mathbf{v}_I$ denotes the restriction of $\mathbf{v}$ to I.

If the entries of A are complex, with i.i.d real and imaginary parts, then with probability at least $1 - e^{-\log^2 n}$ all eigenvectors of A are delocalized in the sense that

$$\min_{I \subset [n],\, |I|=m} \|\mathbf{v}_I\| \geq \frac{m}{n \log^C n} \|\mathbf{v}\|$$

for all $m \in [\log^C n, n]$.

Comparing with related results, in the range $m \in [\log^{C'} n, n/\log^{C'} n]$ in the i.i.d setting and with weaker probability estimates, our bounds on $\|\mathbf{v}_I\|$ strengthen an earlier estimate $\min_{|I|=m} \|\mathbf{v}_I\| \geq c(m/n)^6 \|\mathbf{v}\|$ obtained in [2], and bounds $\min_{|I|=m} \|\mathbf{v}_I\| \geq c(m/n)^2 \|\mathbf{v}\|$ (in the real setting) and $\min_{|I|=m} \|\mathbf{v}_I\| \geq c(m/n)^{3/2} \|\mathbf{v}\|$ (in the complex setting) established in [1].

As the case of real and complex Gaussian matrices shows, our bounds are optimal up to the polylogarithmic multiples. We derive stronger estimates without polylogarithmic error for null vectors of real $(n-1) \times n$ random matrices.

This is a joint work with Konstantin Tikhomirov.

References

[1] K. Luh and S. O'Rourke, Eigenvector Delocalization for Non-Hermitian Random Matrices and Applications, arXiv:1810.00489

[2] M. Rudelson and R. Vershynin, No-gaps delocalization for general random matrices, Geom. Funct. Anal. **26** (2016), no. 6, 1716–1776. MR3579707