HANSON-WRIGHT INEQUALITY IN BANACH SPACES

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The Hanson-Wright inequality states that for any sequence of independent mean zero α-subgaussian random variables $X_1 \dots, X_n$ and any symmetric and real-valued matrix $A = (a_{ij})_{i,j\leq n}$ one has

$$\mathbb{P}\left(\left|\sum_{ij} a_{ij}(X_iX_j - \mathbb{E}(X_iX_j))\right|\right) \leq 2\exp\left(-\frac{1}{C}\min\left\{\frac{t^2}{\alpha^4 \|A\|_{HS}}, \frac{t}{\alpha^2 \|A\|_{op}}\right\}\right). \quad (1)$$

This inequality has found numerous applications in high-dimensional probability and statistics, as well as in random matrix theory. However in many problems one faces the need to analyze not a single quadratic form but a supremum of a collection of them or equivalently a norm of a quadratic form with coefficients in a Banach space. While in the literature there are inequalities addressing this problem, they are usually expressed in terms of quantities which themselves are troublesome to analyze. Our main objective is to find an analogue of (1) in the vector case (i.e when the coefficients are from the Banach space), which can be applied more easily and is of optimal form. The main step in our proof is to derive upper bounds for moments of $S = \sum_{ij} a_{ij} g_i g_j$ (Gaussian chaos of order 2) where $(a_{ij})_{i,j\leq n}$ is a symmetric matrix with values in a Banach space $(F, \|\cdot\|)$. As it turns out our moment estimate can be reversed in a certain class of Banach spaces (including L_q-spaces).

The talk will be based on joint work with Rafał Latała and Radosław Adamczak [1].

References

[1] Radosław Adamczak, Rafał Latała and Rafał Meller, *Hanson-Wright inequality in Banach spaces.* arXiv:1811.00353.