PHASE TRANSITION FOR THE INTERCHANGE AND QUANTUM HEISENBERG MODELS ON THE HAMMING GRAPH

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My talk will be about the cycle structure in models of random permutations generated by random transpositions. One of the motivations for considering these models is the study of the quantum Heisenberg ferromagnet from statistical physics. A particular example of such a model is the interchange process on the complete graph, investigated before e.g., in well-known works by Tóth ([3]) and Schramm ([2]).

In our work, building on previous results by Miłoś and Şengül ([1]), we study similar processes (the interchange process and the cycle-weighted interchange process with parameter $\theta > 0$) on the 2-dimensional Hamming graph $H(2, n)$. We show that in these models the cycle structure of permutations, similarly as on the complete graph, undergoes a *phase transition* – when the number of transpositions defining the permutation is $\leq cn^2$, for small enough $c > 0$, all cycles are microscopic, while for more than Cn^2 transpositions, for large enough $C > 0$, macroscopic cycles emerge with high probability.

In particular for the interchange process we pinpoint exactly the critical time of the phase transition. Our results imply also the existence of a phase transition in the quantum Heisenberg ferromagnet on $H(2, n)$, namely for low enough temperatures spontaneous magnetization occurs, while it is not the case for high temperatures.

Thus we obtain the first known example of a graph with nontrivial geometry for which one can show the existence of such a phase transition. We expect that our approach might prove useful in making progress on the celebrated conjecture due to Tóth on the existence of phase transition for magnetization in the Heisenberg model on $\mathbb{Z}^d$.

Joint work with Radosław Adamczak and Piotr Miłoś.

References

[1] Piotr Miłoś and Bati Sengul, *Existence of a phase transition of the interchange process on the hamming graph, arxiv:1605.03548*, accepted to EJP, 2016.

[2] O. Schramm, *Compositions of random transpositions*, Selected Works of Oded Schramm (2011), 571–593.

[3] Bálint Tóth, *Improved lower bound on the thermodynamic pressure of the spin 1/2 heisenberg ferromagnet*, Letters in Mathematical Physics **28** (1993), no. 1, 75–84.