NUMBER OF ISOMORPHIC COPIES OF A GIVEN GRAPH IN A RANDOM GRAPH

GRZEGORZ SERAFIN (NICOLAS PRIVAULT)

Stein-Chen method and Malliavin calculus on some special probability spaces are used to obtain new result for subgraph counts in random graphs.

Let $\mathbb{G}_n(p)$ denote the binomial Erdös-Rényi random graph constructed by independently retaining any edge in the complete graph K_n on n vertices, with probability $p \in (0,1)$. For a fixed graph G we denote by N_n^G the number of subgraphs of $\mathbb{G}_n(p)$ that are isomorphic to a fixed graph G and by

$$\tilde{N}_n^G := \frac{N_n^G - \mathbb{E}[N_n^G]}{\sqrt{\mathrm{Var}[N_n^G]}}$$

we denote its normalization. Our aim is to estimate rate of convergence of $\tilde{N}_n^G$ to the standard normal distribution $\mathcal{N}(0,1)$. It is shown in [1] that

$$d_W(\tilde{N}_G, \mathcal{N}) \leq C_G \left((1-p_n) \min_{\substack{H \subset G \\ e_H \geq 1}} n^{v_H} p^{e_H} \right)^{-1/2}, \tag{1}$$

where d_W denotes the Wasserstein distance given by

$$d_W(X,Y) := \sup_{h \in \mathrm{Lip}(1)} |\mathbb{E}[h(X)] - \mathbb{E}[h(Y)]|,$$

for two random variables X and Y. However, more natural is the Kolmogorov distance d_K defined as

$$d_K(X,Y) := \sup_{x \in \mathbb{R}} |P(X \leq x) - P(Y \leq x)|.$$

During the talk, we will show that the Kolmogorov distance may be estimated by the same bound as in (1). So far, it was only proved for triangles. Additionally, we introduce a class of graphs G for which the minimum in (1) simplifies to the minimum of only to terms. Finally, analogous results for graphs with weights will be presented.

References

[1] A. D. Barbour, M. Karoński, A. Ruciński (1989). A central limit theorem for decomposable random variables with applications to random graphs. *J. Combin. Theory Ser. B*, 47(2) 125–145.

[2] N. Privault, G. Serafin (2019). Normal approximation for sums of discrete U-statistics - application to Kolmogorov bounds in random subgraph counting. Initially accepted in *Bernoulli*

[3] N. Privault, G. Serafin (2018). Normal approximation for the weights of subgraphs in the Erdös-Rényi random graph. *preprint*