REGULARITY OF SOLUTIONS TO THE STOCHASTIC EQUATION X=AX+B WITH TRIANGULAR MATRICES

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Consider an upper triangular random $d \times d$ matrix with nonnegative entries

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ 0 & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{dd} \end{pmatrix}.$$

Assume that there are $\alpha_1, \ldots, \alpha_d$ such that $\mathbb{E}A_{ii}^{\alpha_i} = 1$ for $i = 1, \ldots, d$. Under additional mild assumptions on $\mathbf{A}$ and on a random vector $\mathbf{B}$, the equation

$$\mathbf{X} \stackrel{d}{=} \mathbf{AX} + \mathbf{B} \tag{1}$$

has a unique solution $\mathbf{X}$. It was recently proved in [3] that if $\alpha_i \neq \alpha_j$ for $i \neq j$, then there are $\tilde{\alpha}_1, \ldots, \tilde{\alpha}_d$ such that

$$\lim_{t \to \infty} t^{\tilde{\alpha}_i} \mathbb{P}(X_i > t) = C_i, \quad i = 1, \ldots, d$$

and the constants C_i are positive.

In this talk I will discuss the same problem in a generalized setting, when we accept that $\alpha_i = \alpha_j$ for $i \neq j$. This regime is somehow complementary to those of [2], [1] and [3]. The main result is the following.

Theorem 1. *Assume that* $\mathbf{A}$ *is an upper triangular random* $d \times d$ *matrix,* $\mathbf{B} \in \mathbb{R}^{\mathbf{d}}$ *is a random vector and some mild assumptions on their laws are satisfied. Then there are* $r(1), \ldots, r(d)$ *such that*

$$t^{\tilde{\alpha}_i} \mathbb{P}(X_i > t) = O\Big((\log t)^{r(i)}\Big) \quad \text{as } t \to \infty.$$

References

[1] Damek, E., Zienkiewicz, J. (2017) Affine stochastic equation with triangular matrices. *Journal of Difference Equations and Applications* published online, 1–23.

[2] Kesten, H. (1973) Random difference equations and renewal theory for products of random matrices. *Acta Math.* **131**, 207–248.

[3] Matsui, M., Świątkowski, W. (2019) Tail indices for AX+B recursion with triangular matrices. *Submitted.*