INTERFACES IN PLANAR ISING AND POTTS MODELS: A REVIEW

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I will provide a review of rigorous results about statistical and geometrical properties of the interface between two equilibrium phases in the planar Ising and Potts models. Among the topics to be discussed:

- exact results on the profile of expected magnetization in the 2d Ising model;
- microscopic structure of the interface in Ising and Potts models;
- invariance principle for Ising and Potts interfaces and regularity properties of the equilibrium crystal shapes;
- pinning of Ising and Potts interfaces by a row of modified coupling constants;
- effect of an external magnetic field on the interface: critical prewetting in the Ising model.

The results I will present have been obtained over several decades (from the 1970s to the present day), with contributions by many people. Techniques used in the earliest works range from exact computations, relying on the integrability of the planar Ising model, to various perturbative approaches. Most of the more recent works rely on the nonperturbative Ornstein–Zernike theory, that allows to construct a coupling between the interface and a suitable directed random walk (usually in an external potential), combined with an analysis of the latter process.