A DIMENSION-FREE ESTIMATE ON $\ell^2(\mathbb{Z}^d)$ FOR THE DISCRETE DYADIC MAXIMAL FUNCTION OVER THE EUCLIDEAN BALLS.

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The topic of dimension-free L^p estimates for Hardy–Littlewood maximal functions over convex bodies in $\mathbb{R}^d$ had been mainly developed in the '80 and '90 in the work of Stein, Bourgain, Carbery, and Müller. The interest in the topic has been recently renewed due to recent progress by Aldaz 2011 and Bourgain 2014.

Last year, together with J. Bourgain, M. Mirek, and E.M. Stein, we initiated the study of dimension-free bounds for maximal functions in discrete contexts, i.e. with $\mathbb{R}^d$ replaced by $\mathbb{Z}^d$. In this talk I am going to present a positive result for discrete Euclidean balls. Namely, I shall justify that the dyadic maximal function is bounded on $\ell^2(\mathbb{Z}^d)$ with a bound independent of the dimension d. The proof is based on various dimension-free estimates for exponential sums (Fourier transforms). Probabilistic techniques on permutation groups form a central ingredient for handling these sums. Additionally, when treating suprema over small radii we employ a combinatorial argument that relies on Krawtchouk polynomials.

The talk is based on joint work with J. Bourgain, M. Mirek, and E.M. Stein.