Aniosotropic integrands in the theory of varifolds

Varifolds are geometric objects with singularities that appear naturally in variational problems involving minimisation of functionals defined on smooth manifolds. The *area functional*, i.e., the functional that evaluates to the total measure of a manifold, is the most known and well studied example. The celebrated *Plateau problem* is about finding a surface with least area among those having a given boundary.

Since there is no restriction on the topology of the minimiser one cannot parameterise all competitors with a single manifold. Moreover, one should not assume that the minimiser carries some additional algebraic structure, e.g., that it is orientatable or, more generally, that it is an integral current with coefficients in a fixed abelian group. Varifolds seem to be the most versatile model.

Lectures will cover the following topics.

1. Basic notation and definitions concerning varifolds.
2. The first variation with respect to an arbitrary integrand.
3. Various notions of ellipticity for geometric integrands.
4. Summary of known existence and regularity results for minimisers and critical points of elliptic functionals.
5. Anisotropic integrands in codimension one.

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Sławomir Kolasiński
Instytut Matematyki, Uniwersytet Warszawski
ul. Banacha 2, 02-097 Warszawa, Poland
`s.kolasinski@mimuw.edu.pl`